Anti-Fragile Decision-Making: tHink beYonD Robustness And resilience (HYDRA)

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Abstract

Humanity continues to struggle with various crises. Recent wars, energy and economic crises, pandemics, and climate change underscore the vulnerability and fragility of decisions made based on unreliable predictions, which often overlook plausible but less probable scenarios, and disregard the potential consequences of those decisions on all stakeholders. The primary aim of this paper is to support decision-makers in being well-prepared for a continuously changing future and making sustainable, anti-fragile decisions to create a better world. We renewed the perspective on decision-making under uncertainty to think beyond robustness and resilience, and become anti-fragile — to look for opportunities to benefit from variability (e.g., adapting to online shopping during a pandemic). Indeed, we conceptualize antifragility and design a generic modular architecture for decision support tools and anti-fragile decision-making. Hence, we (1) comprehensively review, classify, and discuss the wide, multidisciplinary literature in a systematic way to identify effective properties and attributes of anti-fragile systems and their connections, various existing ways suggested for anti-fragile measurement, the concepts behind them, their limitations, and the required properties of a reasonable measurement for relevant decision-making; (2) propose a proper antifragility measurement that matches the requirement and possible meaningful ways of its usage in anti-fragile decision-making, such as relevant strategy generation and analysis; and (3) design and propose a generic anti-fragile decision-making framework (HYDRA) to determine, analyze, and compare anti-fragile strategies, including adaptive pathways and contingency plans, based on multiple criteria. This novel perspective sheds light on the birth of a new era in decision-making under uncertainty.

keywords: Anti-fragile measurement, Thriving performance, Dynamic adaptation, Deep uncertainty, Multi-criteria decision-making

1 Introduction

Many real-life problems are tied to unpredictable events, particularly those related to the future and human actions (e.g., climate change, natural hazards, socioeconomics, pandemics, military/ cyber-attacks, political crises, and religious beliefs). Here, decisions must be made without complete knowledge of the consequences, all available options, and the future state of the world (referred to as a scenario from now on) Van der Heijden (1996), Shavazipour & Stewart (2021). Even if the consequences of all choices could be identified, the exact realization of the future scenario is unknown. With current technology and knowledge, accurately forecasting future outcomes and transitions is impossible due to various external uncertainties and the lack of historical data. Therefore, many future-oriented decisions that have been made based on a trend analysis of some (unreliable) predictions for an average or the most probable realization of the future often fail due to careless consideration of other plausible scenarios. This situation, *where the likelihoods of plausible future scenarios are unknown, or the stakeholders cannot agree upon one, is called **deep uncertainty*** Lempert et al. (2003), Walker et al. (2013).

In decision-making under deep uncertainty (DMDU), the context experts cannot wholly identify all outcomes, boundaries, probability distributions, or related preferences of the system (Lempert et al. 2003, Walker et al. 2013). We may, for example, specify plausible climate change scenarios based on different levels of CO_2 emissions, temperature, and precipitation, or we can identify the energy dependency sources of a country, plausible scenarios of losing those sources, and replacement options. Yet, exact estimations of their occurrence probabilities and accurately forecasting their impacts are impossible because of many external influential but uncertain factors such as governments and society's adherence to the (inter)national agreements, long-term maintenance plans and their actual impacts, natural disasters, world security crisis, future technological inventions, and social transformations. Additionally, the performance of a decision varies in different scenarios, making the decision-making process overly complex. Adding surprises to such a complex decision problem can even make it chaotic.

Most of the available decision-support tools and trending machine learning (ML) algorithms focus on a lower level of uncertainty with known probability distributions, namely **mild** uncertainty (Shavazipour & Stewart 2021), and unreliable historical-based predictions. Nonetheless, any unexpected shocks or disruptions may lead to failure. They are also often slow to benefit from new opportunities. Moreover, some naive impressions have misled many scientists into the *blind* usage of ML algorithms without checking their suitability, data quality, and level of uncertainty. *There is a need* for advanced decision-support tools in such complex decision problems that do not entirely rely on historical data but also consider various kinds of uncertainty and the consequences of the decisions (Shavazipour et al. 2020, Shavazipour & Stewart 2026).

Robust (Bertsimas & Sim 2004, Lempert et al. 2003, 2006) and *resilient* (Christopher & Peck 2004) solutions have been introduced to relieve the harmful effects of deep uncertainty. A robust solution is less sensitive to variability (Lempert et al. 2006, Shavazipour et al. 2021a) while a resilient solution can quickly recover from variability resulting from deep uncertainty (Brown et al. 2020). Using scenarios can help us visualize, plan, and interpret different possibilities of future states without assuming any probability distributions (Durbach & Stewart 2012, Shavazipour et al. 2021b). In this sense, robust solutions are those that stay acceptable (or are not vulnerable) in a wide range of future scenarios (Lempert et al. 2006, Shavazipour & Stewart 2026), and resilient solutions represent the contingency plans that apply when a less probable (sometimes called non-nominal) scenario manifests. Indeed, we should minimize the negative impacts of plausible shocks and stress in the future, also known as *futureproofing*. However, most robust solutions are known as conservative solutions (the cost of robustness) (Bertsimas & Sim 2004), and resilient solutions are costly and may fail if we pass the tipping points (Shavazipour & Stewart 2021). Recently, dynamic robust and multi-stage multi-scenario approaches have been developed in the literature to slightly reduce the cost of robustness (Bertsimas et al. 2019, Haasnoot et al. 2013, Shavazipour et al. 2020, Shavazipour & Stewart 2026, Shavazipour, Kwakkel & Miettinen 2025). Nevertheless, there is still a price to pay for robustness, and the danger of fragility in the case of surprises.

The main open question is **"how to go beyond robustness and resiliency and be anti-fragile"**— i.e., prepare for rare events (**Black Swans**) without recourse to cause (Derbyshire & Wright 2014). This needs a *paradigm shift* in thinking and serious planning, ensuring long-term resiliency, sustainability, and robust transformations. More specifically, there is an acute need for *novel patterns* in decision-making and operations research *to define, measure, model, support anti-fragile thinking*, and identify *anti-fragile strategies* for better decision-making. Our *primary intention* in this study is to answer *"how to design a modular structure for anti-fragile decision-making and develop building blocks for a practical decision support tool to support decision-makers in complex decision problems"*.

Objectives: To *open a new era* in Operations Research and decision-making under deep uncertainty to think beyond robustness and resilience against slight deviations from the average or the most probable values and, instead, to look for opportunities to benefit from variabilities by conceptualizing *anti-fragility* (Taleb 2012) in multi-criteria decision-making and designing a general modular architecture for decision support and anti-fragile decision-making. Indeed, *an antifragile system* implies consideration of *rare events* and *the ability to change* (adaptive pathways). The goal is not to find the Black Swans, but to determine where they are not (Levin et al. 2014), to handle them with current or initial strategies. Then, to concentrate on possible (or plausible) Black Swans, perhaps convert them to a more simplified and manageable version (i.e., *Gray Swans*), and identify adaptations for those situations that challenge performance.

To reach this aim, we

- (i) comprehensively review, classify, and discuss the wide, multidisciplinary literature in a systematic way to identify effective properties and attributes of anti-fragile systems and their connections, various existing ways suggested for anti-fragile measurement, the concepts behind them, their limitations, and the required properties of a reasonable measurement for relevant decision-making (**RQ1**);
- (ii) propose a proper antifragility measurement that matches the requirement and possible meaningful ways of its usage in anti-fragile decision-making, such as relevant strategy generation and analyses (**RQ2**); and
- (iii) design and propose a generic anti-fragile decision-making framework (HYDRA) to determine, analyze, and compare anti-fragile strategies, including adaptive pathways and contingency plans (a chain of decisions exploring adapting options in different stages of a planning horizon) to be not only resilient to disruptions and possible transformations but also improve overall performances in a long-term horizon, based on multiple criteria (**RQ3**).

The decision support tools developed based on the above concepts, definitions, framework, and models should also (iv) *interactively* involve decision-makers in all stages of the decision-making process (RQs 2&3), and (v) consider decision-makers' *preferences* and the *consequences* of alternative decisions to make more *sustainable* and *anti-fragile* decisions (RQ3).

2 Literature review (RQ1)

This section presents a comprehensive review of the extensive and multidisciplinary literature related to systems' antifragility, its properties and features, possible measurement methods, and applications in decision-making, which helps us answer RQ1 and paves the way for answering RQs 2 and 3.

2.1 Aim and research questions

As briefly mentioned earlier, the primary purpose of this review is to identify and extract properties of anti-fragile systems, including the attributes or features of anti-fragile strategies, relevant measurements, and their application in decision-making problems. Therefore, we can investigate their connections, classify them, and compare their pros and cons to propose proper antifragility measurement and possible ways of its usage in anti-fragile decision-making, leading to the proposal of guidelines and a generic framework for antifragile decision-making. These sub-research questions for the literature review can be summarized as follows:

SRQ1: How is the term anti-fragility defined in different disciplines?

SRQ2: How is anti-fragility measured? Is there any analytical way of measurement?

SRQ3: Is there any guideline or model already proposed to be used in decision-making?

2.2 Systematic review method

To review the literature, we follow the PRISMA guidelines for a systematic review (Moher et al. 2009), including the following Phases and steps, as demonstrated in Figure 1:

Phase 1. Identification: The systematic literature review started by identifying relevant articles to read in January 2025.

Step 1: We first conducted the search by using keywords among journal papers in the Web of Science and Scopus databases. The search keywords are described in Table 1.

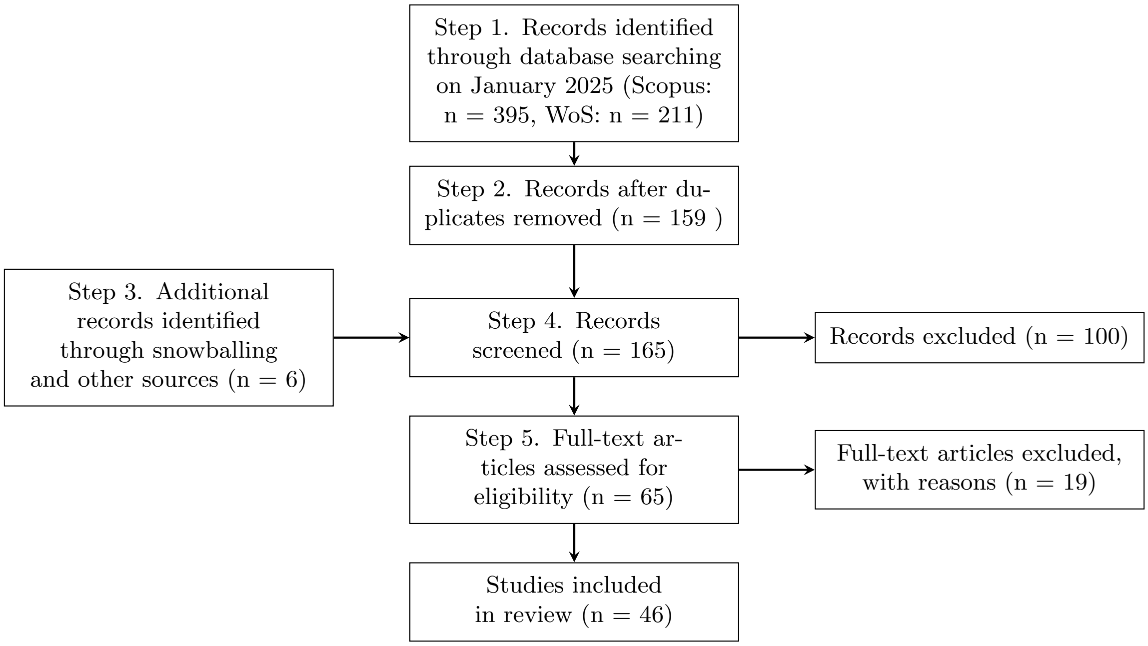

Figure 1: The PRISMA diagram for the literature review.

Databases	Keywords
Scopus	TITLE-ABS-KEY (“anti-fragil*” OR “antifragil*”) AND (LIMIT-TO (LANGUAGE, “English”)) AND (LIMIT-TO (DOCTYPE, “ar”) OR LIMIT-TO (DOCTYPE, “cp”) OR LIMIT-TO (DOCTYPE, “ch”) OR LIMIT-TO (DOCTYPE, “re”) OR LIMIT-TO (DOCTYPE, “bk”))
WoS	ALL (“anti-fragil*” OR “antifragil*”) AND refile by (LIMIT-TO (LANGUAGE, “English”)) AND (LIMIT-TO (DOCTYPE, “article”) OR LIMIT-TO (DOCTYPE , “review article”) OR LIMIT-TO (DOCTYPE, “early access”) OR LIMIT-TO (DOCTYPE, “proceeding paper”))

Table 1: Search keywords used to identify articles for the literature review.

Step 2: Checking the paper's information and removing duplicates.

Phase 2: Screening and Exclusion

Step 3: Adding 6 other papers found through other resources, such as papers known/read before and not appearing in the initial search, and identification through snowballing—i.e., backward (reviewing the references of the articles) and forward search (reviewing articles that refer to the articles), to ensure identifying all related publications.

Step 4: Then, the titles and abstracts of all the identified papers are screened, and irrelevant papers are removed based on the exclusion criteria. We also quickly review the conclusion and other sections of some papers to ensure that no relevant publications have been omitted when the title and abstract were not clear enough to make a decision. The exclusion criteria were: (1) too conceptual papers without pointing out any specific features, measurements, suggested guidelines, or models; (2) very specific focus on some application domains, such as cyber-security or software engineering, without any measurement, guidelines, or models. Most of the papers removed in this phase were only utilizing keywords in their abstract without any descriptions or relevance to the study.

Phase 3: Inclusion

Step 5: Then all the 65 remaining publications have been fully read carefully to evaluate their eligibility and select the most relevant papers with potential to answer the literature review questions (SQR1-3). In this phase, for the inclusion criteria, we mainly consider: (1) any definition, property, attributes, criteria, or features related to antifragility; (2) any proposal for measuring fragility or antifragility; (3) relevance to decision-making and anti-fragility concepts; or (4) any qualitative guidelines or models for antifragility with potential generalization.

Therefore, a total of 46 papers are included in this review part. The selected papers and their perspectives for antifragility properties/attributes/criteria/features, possible concept or model for measurement, and the number of criteria/objectives for antifragility are portrayed in Table 2. The classification of properties and attributes of antifragility, as extracted from these review papers, along with details of existing measurements and the relevance and importance of various antifragility criteria, will be discussed in the following sections. Before proceeding, we will conduct a brief overall review of the selected papers.

Papers	Anti-fragile properties/features	(Anti-)fragile measure	# objectives/criteria
Taleb & Douady (2013)	convexity of payoffs	right tail payoff sensitivity	-
Tolk & Johnson IV (2013)	agility	-	-
Johnson & Gheorghe (2013)	absorption, redundancy, adding low level stress, non-monotonicity, requisite variety, emergence, coupling, balancing constraints vs. freedom, entropy, efficiency vs. risk	antifragility analysis framework	10
Derbyshire & Wright (2014)	optionality, barbell strategies, redundancy, hormesis, and small-scale experimentation	-	-
Levin et al. (2014)		convexity-based approach	
Aven (2015)	most probable positive consequences	-	-
Gorgeon (2015)	simplicity, skin in the game, redundancy optionality, injecting randomness, decentralization	outcomes convexity	-
Meissner & Wulf (2015)	regular	-	-
Fang & Sansavini (2017)	performance improvement in shocks	-	2
Abou Kasm et al. (2017)	performance improvement	-	1
Ghasemi & Alizadeh (2017)	absorption, redundancy, stress Starvation non-monotonicity, requisite variety, emergence, uncoupling	fuzzy antifragility index (FAI)	7
Taleb (2018)	convexity of payoffs	right tail payoff sensitivity	-
Hysa et al. (2018)	ability to reinvest profits, adapt to crises, foster social cohesion	-	-
Kiefer et al. (2018)	metastability (a balance between stability and flexibility)	phenotypic plasticity	-
Pineda et al. (2019)	stability, flexibility	satisfaction change	-
Kim et al. (2019)	complexity, perturbation	perturbation * satisfaction change	-
Kim et al. (2020)	complexity, perturbation	perturbation * satisfaction change	-
de Bruijn et al. (2020)	regular	performance variation from the Mean	-
Ramezani & Camarinha-Matos (2019)	-	-	-
Ramezani & Camarinha-Matos (2020)	-	-	-
Equihua et al. (2020)	robustness, redundancy, dynamic adaptability	convexity of payoff functions	-
Moudi et al. (2020)	reliability, resiliency, vulnerability	average of 3 features	3
Blečić & Cecchini (2020)	optionality	-	-
Nikookar et al. (2021)	dynamic and fluid		
Bravo & Hernández (2021)	diversification, vertical integration, optionality, flexibility, sustainability	Altman's Z-Score model	-
Botjes et al. (2021)	redundancy, diversity, self-organization, modularity	-	-
Mahata et al. (2021)	-	liquidity balance/operating costs	
Mardaras et al. (2021)	optionality, dispersion, heuristic experimentation, learning from failure	-	-
Munoz et al. (2021)	redundancy, flexibility, proactive, reactive	-	-
López-Corona & Kolb (2021)		entropy production	-
Goodwill et al. (2022)	regular	-	4
Axenie et al. (2022)	local convex response	-	5
Nassehi et al. (2022)	regular	-	-
Chenaru et al. (2022)	adapt and improve under stress	-	2
Sandhya & Chatterjee (2022)	to remain operational while being exposed to stress/shocks	-	1
Niemeier (2022)	failure is often defined by exceeding limits	fragility curve/surface (failure probabilities)	-
Coppitters & Contino (2023)	more upside potential	skewness	3
Lotfi, Mehrjardi, MohajerAnsari, Zolfaqari & Afshar (2023)	viability	-	1
Lotfi, Hazrati, Ali, Sharifmousavi, Khanbaba & Amra (2023)	viability	-	1
Alatorre et al. (2023)	-	satisfaction * perturbation	-
Adelhart Toorop et al. (2023)	adaptiveness, optionality, redundancy, autonomy, bottom-up innovation	-	
Gotardelo & Goliatt (2024)	better performance in volatile markets	Conditional Volatility Index (CVIX)	6
Lotfi et al. (2024)	viability (resilience, sustainability, agility)	-	1 (3)
Vazquez-Noguerol et al. (2024)	agility, adaptability and flexibility	-	1
Li et al. (2024)	better average results than adjusting the current scenario	antifragility analysis algorithm (AAA)	1
Renigier-Bilozor et al. (2024)	elasticity/flexibility/optionality, less centralized planning, proactive engagement	-	-

Table 2: Caption

2.3 Overall review of the included articles

This section provides a brief overview of the literature review, which includes the papers identified through the systematic method described above. We cover the definitions and suggested measurement approaches, along with their applications in relevant decision-making problems, as they relate to answering our review sub-research questions (SRQ1-3).

2.3.1 How is the term anti-fragility defined in different disciplines? (SRQ1)

Taleb defined, for the first time, the term *antifragility* as the opposite of fragility or vulnerability, when a system grows, instead of falling, in the face of shocks raised by volatility or stressors (Taleb 2012). Different from other perspectives on withstanding harm (robustness) or recovering to regular performance (resiliency), they tied antifragility to a *convex* response to shocks and changes, resulting in more upside (benefits) than downside (losses) performance. They highlighted the advantages of anti-fragile thinking by comparing the ease of identifying fragilities and vulnerabilities with designing a plan that withstands all the shocks. As an example, they also highlight the fragility of the economic system by referencing the 2007 economic crisis. Additionally, various crises have been observed recently, including the COVID-19 pandemic and the ongoing wars in Ukraine and the Middle East. Following this, having a more upside than downside (convex) response to shocks (or random events) is considered *antifragility* in various applications, such as network of infrastructure (Feng et al. 2017), airline industry (Abou Kasm et al. 2017), currency production (Ghasemi & Alizadeh 2017), medicine and medical risk management (Taleb 2018), sports and athletic training (Kiefer et al. 2018), disaster management (Ramezani & Camarinha-Matos 2020), cancer therapy (Axenie et al. 2022), manufacturing and production systems (Nassehi et al. 2022, Chenaru et al. 2022), water systems (Goodwill et al. 2022, Moudi et al. 2020), energy systems (Johnson & Gheorghe 2013, Bravo & Hernández 2021, Sandhya & Chatterjee 2022, Niemeier 2022, Coppitters & Contino 2023), supply chain (Vazquez-Noguerol et al. 2024, Lotfi, Mehrjardi, MohajerAnsari, Zolfaqari & Afshar 2023), and finance (Gotardelo & Goliatt 2024). In this regard, having an *average* response across all scenarios greater than the response from the average scenario is also considered a form of antifragility (Levin et al. 2014, Kim et al. 2019, 2020, de Bruijn et al. 2020, Equihua et al. 2020, Mahata et al. 2021).

Additionally, Aven (2015) connects antifragility to risk management and highlights that the primary purpose of risk management is to evaluate uncertainties and support decision-making, rather than accurately estimating the probabilities of rare events (or *Black Swans*). They claimed that all systems are anti-fragile to some extent, and we can enhance most systems by increasing the positive risks and mitigating the negative ones, while there is no actual perfect robust system. Identifying Black Swans (unknown-unknown or the events that are not in the event list, often because of very low occurrence probability (Aven 2015)) and evaluating possible consequences in various scenarios, rather than predicting their occurrence chance and where they are not, can help to benefit from Black Swans while limiting harm (Levin et al. 2014, Gotardelo & Goliatt 2024). Considering Black Swans and extreme events as fat-tailed response distributions (particularly right-tail skewness) is also in line with Taleb's definitions and investigated by several authors to explore connections to some statistical methods—when there is no probabilistic independence, complexity related to fat-tail responses, and fat-tails with antifragility (Taleb & Douady 2013, Taleb 2018, Equihua et al. 2020). Besides, handling disruptions, as a kind of surprise, and related concepts and approaches in business ecosystems are reviewed to highlight promising approaches to enhance resiliency and antifragility (Ramezani & Camarinha-Matos 2019, 2020). Their conclusion emphasizes vulnerability reduction and transforming disruptions into business opportunities.

The shift from pure predictions and causality to identifying fragility and possible adaptations for performance growth in the face of Black Swans, which originated in Taleb's concept of antifragility, is raised as a suitable strategy for handling deep uncertainty in any organization (Derbyshire & Wright 2014). In this anti-fragile perspective, reducing fragility by identifying adaptations for mitigation and improvement is prioritized (Equihua et al. 2020, Nikookar et al. 2021). To achieve such an improvement, concentrating on long-term effects and consequences rather than short-term gains is also highlighted as an effective approach (Fang & Sansavini 2017, Hysa et al. 2018). Derbyshire & Wright (2014) also introduced *optionality*, *redundancy* (backup support), *hormesis* (low-dose or stress as an antidote or adaptation), *small-scale experiments*, and *barbell strategies* (mostly invest in no-/low-risk low-gain and a minor portion in high-risk high-gain (Taleb 2012)) as promising ways for improving antifragility, and proposed a qualitative method for investigations and evaluations in related decision-making. Similar guidelines for considering effective antifragility attributes and features in analyses and decision-making are also suggested by various authors, as reviewed in the second column of Table 2, and thoroughly discussed and classified in Section 3. For example, Gorgeon (2015) suggested similar attributes for anti-fragile information systems such as *simplicity*, *redundancy*, *optionality*, *skin-in-the-game* (directly affected by consequences), *decentralization*, and *injecting randomness*. Meissner & Wulf (2015) integrated some antifragility attributes and principles with the backward logic method (Wright & Goodwin 2009) to qualitatively investigate the consequences of possible core strategies and relevant adaptation strategies in various scenarios. Blečić & Cecchini (2020) emphasized keeping the *optionality* attribute to allow adaptations and possible ways of improvement and benefits in case of surprises. Diversification, vertical integration, optionality, flexibility, and sustainability are mentioned as key resilience attributes in organizations. However, this raised the need for further study to identify common attributes of anti-fragile organizations and to develop antifragility measurement (Bravo & Hernández 2021).

2.3.2 How is anti-fragility measured? Is there any analytical way of measurement? (SRQ2)

While a broad range of properties and attributes is suggested for antifragility, with some potential conflicts, evaluating and comparing the antifragility of strategies across various scenarios requires a sound measurement. Various studies have introduced different methods for measuring antifragility, based on performance variations that analyze and compare positive versus negative changes. Reviewing the literature shows the existence of both (semi)qualitative (subjective/expert-judgment-based) (Aven 2015, Ghasemi & Alizadeh 2017, Li et al. 2024) and quantitative and analytical (Taleb & Douady 2013, Pineda et al. 2019, Kim et al. 2019, 2020, de Bruijn et al. 2020, Equihua et al. 2020, Moudi et al. 2020, Mahata et al. 2021, Niemeier 2022, López-Corona & Kolb 2021, Alatorre et al. 2023, Gotardelo & Goliatt 2024) measurement approaches. Some authors are also mainly focused on a particular domain or problem and developed specific measurements. For instance, Bravo & Hernández (2021) utilized Altman's Z-score financial equation (Altman 1968) to evaluate production growth in some companies in Latin America, the US, and Canada, which is suitable for resilient measurement and can be combined with other metrics for anti-fragile measurement. Kiefer et al. (2018) also measure antifragility through phenotypic plasticity, quantified as the area under the fitness curve, in performance levels of athletes, demonstrating adaptation capacity and growth in response to environmental changes. A more comprehensive review, classification, and comparison of existing antifragility measurements are discussed in Section 4, before examining their limitations (Section 4.4) and proposing a novel antifragility measurement to address those limitations, along with potential applications in anti-fragile decision-making in Section 5.1 and 5.2, respectively.

2.3.3 Is there any guideline or model already proposed to be used in decision-making? (SRQ3)

Besides identifying properties suitable for antifragility evaluation and measurement, it is necessary to integrate various elements that support the generation of anti-fragile strategies and possible adaptations, as well as relevant comparison tools and analyses, for informed anti-fragile decision-making. Many recent studies have focused on specific strategies to enhance antifragility in various domain-specific problems. Renigier-Bilozor et al. (2024) conceptualize antifragility in the property tax system for regional development and sustainability by supporting the involvement of local stakeholders in decision-making processes. Mahata et al. (2021) developed a stock price model to simulate stock price changes during a shock, using data related to COVID-19, to investigate the relationship between price recovery and financial antifragility and support investment decision-making.

Five criteria are considered by Axenie et al. (2022) to evaluate the therapy controllers (strategies) generated by pulsed, robust, optimal, and anti-fragile control models—i.e., time to tumor elimination, maximum drug concentration, total drug administered, maximum tumor cell count, and minimum normal cell count. In a simulated experience, they demonstrated the positive potential of their proposed anti-fragile controltheoretic approach in comparison with robust and optimal control models, anticipating future variability and exploiting drug dosing volatility to achieve a faster reduction in tumor burden, while exploring the trade-off between drug concentration and collateral damage to normal cells.

Model-based supports for decision-making are also explored by numerous studies. Vazquez-Noguerol et al. (2024) investigate the role of collaborative networks in transportation and the sharing of relevant information to increase optionality and redundancy in the company's supply chain, leading to more anti-fragile performance. Lotfi, Hazrati, Ali, Sharifmousavi, Khanbaba & Amra (2023) formulate optimization models for healthcare waste management. To consider antifragility, resiliency (extra or flexible capacity), sustainability (emissions, consumption, and risk), and agility (as enhanced by adding blockchain technology), they introduced different sets of constraints to the model. Similarly, they adjusted the optimization model for supply chain network design problems (Lotfi, Mehrjardi, MohajerAnsari, Zolfaqari & Afshar 2023, Lotfi et al. 2024). In all models, they assumed known probabilities of scenarios, conservatism coefficients, and confidence levels in conditional value at risk, reducing the applicability of their model in handling deep uncertainty.

Due to the multiple, potentially conflicting antifragility properties and attributes, some authors have considered analyzing the antifragility of strategies through multiple criteria or objectives. Johnson & Gheorghe (2013) introduced a conceptual anti-fragile simulation model and an antifragility analysis framework for systems facing shocks (or, as they named them, *X-Events*) based on ten criteria (extracted from the list gathered from case studies on systems of systems by Jackson & Ferris (2013)), which were hypothetically tested in a smart grid electrical power system example. They suggested developing a questionnaire to gather experts' and/or stakeholders' opinions on the system's response to stress and quantify the answers on an interval scale for each criterion or an aggregated criterion. As example methods, they mentioned the Delphi method (Linstone et al. 1975, Ishikawa et al. 1993, Rowe & Wright 2001) to aggregate individual answers and the use of fuzzy logic (Zadeh 1965) for quantification. Following these suggestions, seven criteria (see Table 2) are considered by Ghasemi & Alizadeh (2017) to design a questionnaire (with a five-point Likert scale) to elicit weights of each criterion for an antifragility analysis in a currency production case study. They employed an entropy method and triangular fuzzy numbers to calculate an aggregated fuzzy rating and fuzzy average weights for each criterion, as a component of antifragility, to determine a fuzzy antifragility index. Recently, similar conceptualization has been undertaken to utilize experts' judgments for evaluating strategy performance and an antifragility analysis in decision-making using methods in multi-criteria decision analysis (MCDA) by Li et al. (2024).

Further usage of optimization models has been considered by (Fang & Sansavini 2017) to propose a bi-objective optimization model (maximizing flow vs. minimizing total investment costs) with limited investment constraints for the recovery planning of post-disaster infrastructure networks in the power transmission grid of northern Italy, involving the construction of new components rather than the repair of failed ones. They employed the weighted sum method (Gass & Saaty 1955) to solve their bi-objective problem, demonstrating improved functionality with lower expenses compared to the original network. Besides, Abou Kasm et al. (2017) raised three goals (minimizing costs, maximizing revenue, and maximizing customer satisfaction) as companies' objectives to maximize their profit and stay operating in the market. However, they generate strategies by separately optimizing each goal and conducting an analysis to assess their impact in different circumstances, such as unexpected customer delays and/or weather conditions. Then, they used the results to update the optimization models and suggested a modeling framework for updating optimization models as an improvement. Moudi et al. (2020) formulate a three-objective optimization model for fragility evaluation, including reliability (the probability of satisfactory performance—e.g., meeting periodic demand in a water supply system), resilience (the probability of not encountering any shortage), and vulnerability (when experiencing a failure). As a fragility index, they optimized the weighted sum of the average of these three objectives across multiple periods, and compare the results in three different future scenarios: the case of average annual precipitation, as well as slight and extreme cases of drought, corresponding to 10% and 30% drops in rainfall, respectively. As a result, they concluded that increasing the reliability of the system is more effective in enhancing its antifragility compared to the other two objectives in a water supply system with a long-term planning horizon.

The existing trade-off between long-term (such as antifragility) and short-term performances (e.g., capital and operating costs) has also been pointed out by Goodwill et al. (2022). They also raised the point that a lack of sufficient data, unreliable and/or unknowable probabilities in many real-world problems, leads to unclear distributions with skewness (such as Gaussian or Gamma) or fat-tails (such as Cauchy), which heightens the complexity (volatility) and the level of uncertainty. They, instead, emphasize a systematic assessment of anti-fragile opportunities and pilot stress testing and simulations. Regarding such stress testing and simulation, Chenaru et al. (2022) designed a self-improving manufacturing system method for anti-fragile maintenance based on artificial failure injection and analysis to identify possible alternative adaptations. They suggested this anti-fragile maintenance analysis to assess risk by predicting malfunctions and potential maintenance needs, based on learning from artificial errors, and to better prepare anti-fragile responses to Black Swans. They connected the increasing resilience of a system in vulnerable conditions to the multiobjective optimization of efficiency, robustness, and intervention time in designing an anti-fragile system. To investigate the trade-offs between optimality and robustness of the strategies, they suggested using a bi-objective optimization model and employing the weighted sum method to solve a minimization problem that balances the weighted amount of quality and robustness of the solution or strategy (referred to as functional robustness).

Furthermore, Coppitters & Contino (2023) suggested using skewness as an indicator of antifragility (or objective) to optimize upside performance variability while limiting downsides to an acceptable performance level. Their three-objective probabilistic optimization model, called the anti-fragile design optimization framework, incorporates the Upside Potential Ratio (UPR), a reward-risk ratio that accounts for skewed returns, the median (as a measure of average performance), and skewness. Their strategy generation is significantly dependent on the initial distributions because of the assumption of known distributions for input parameters. Multiobjective optimization models have also been utilized by Gotardelo & Goliatt (2024) to consider various antifragility attributes for strategy investment generation in the portfolio selection problem. They then compare the generated strategies with those produced by other models, including the traditional *capital asset pricing model* (CAPM) (Markowitz 1952, Sharpe 1964), and demonstrate the superiority of the strategies generated by models that consider various attributes over the traditional CAPM Mean and Variance model in a US market sample facing crises. Of course, when the objectives change, different strategies or solutions are expected.

The remainder of the paper is allocated for in-depth discussions, classifications, and comparisons of the concepts, attributes, measurements, and approaches proposed in the literature to address the primary research questions and aims of this work. Furthermore, we examine the limitations of existing approaches suggested for antifragility measurements to propose a novel measurement approach and potential ways of antifragility measurement in decision-making, leading to a generic framework for anti-fragile decision-making (to be called HYDRA).

3 The main properties of anti-fragile systems (RQ1-2)

As mentioned earlier, an anti-fragile system is defined by its performance thriving in the long term, despite various shocks. Indeed, while a system must have some degree of long-term withstanding and recovery in facing shocks and stressors, it should also be able to thrive (achieving significantly high positive performance), even in less likely situations, and small/acceptable negative performance, perhaps in some other situations (Aven 2015). As fragility and vulnerability are directly relevant, no (or least) vulnerability is a general characteristic of antifragility.

Besides, different features, criteria, or properties mentioned in the literature can contribute to reaching antifragility, shown in Table 2. Some of them have been gathered through questionnaires from experts and practitioners in various industries (Ghasemi & Alizadeh 2017), while the others are based on practical and academic experiences that showed performance improvements facing shocks (Taleb & Douady 2013, Johnson & Gheorghe 2013, Aven 2015, Equihua et al. 2020, Bravo & Hernández 2021). They might also be used in different ways, such as developing anti-fragile strategies or utilizing them in measuring antifragility. In this section, we discuss those that are common and necessary to decision-making in various fields and potentially

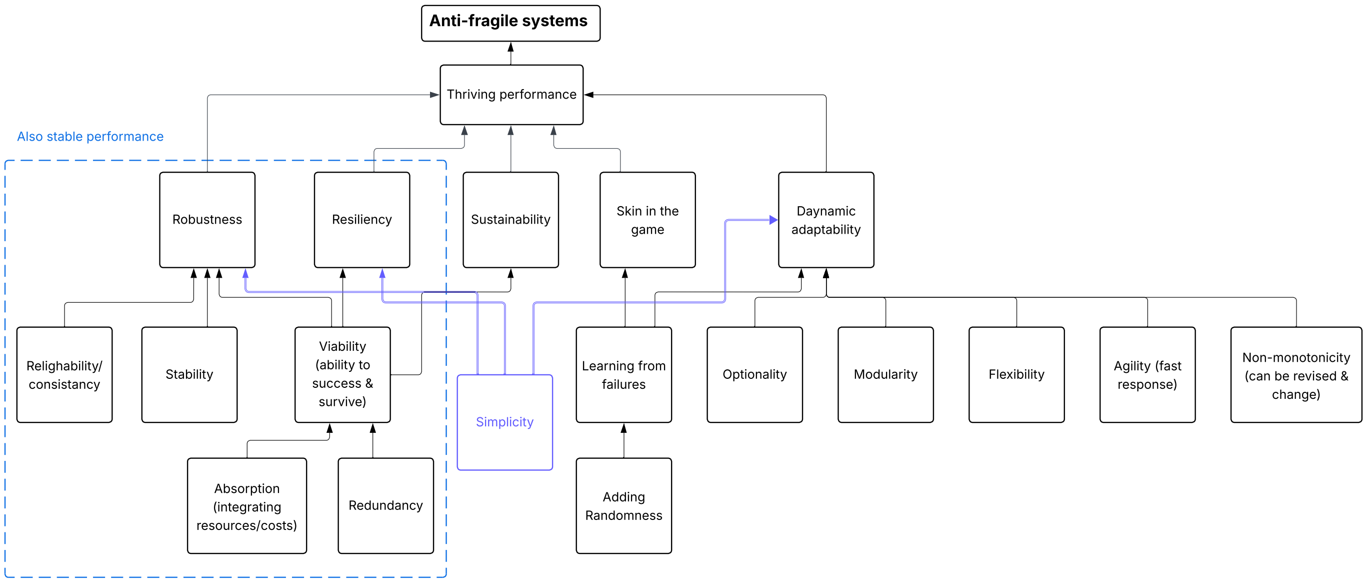

Figure 2: Main properties and attributes of anti-fragile systems.

useful in optimization-supported management and decision-making. Moreover, we focused on how they connect or affect each other to gain better long-term performance under changing and deeply uncertain environments, as summarized in Figure 2.

3.1 Main necessary properties

As shown in Figure 2, *robustness*, *resiliency*, *sustainability*, *skin in the game*, and *dynamic adaptability* are the main properties required for antifragility. Based on Taleb's definition of antifragility, robustness and resilience have different focuses, withstanding shocks or recovering quickly with similar performance levels. However, these two properties are necessary for antifragility in various situations. If a system easily fails when faced with a shock or can hardly recover, achieving better performance is either impossible or costly. Therefore, these two are highlighting the consistency and dependency and are mainly required for an anti-fragile system, although their existence is not always necessary for the other.

Sustainability considers various aspects and seeks to optimize resource use, minimize waste and prevent depletion, to ensure enduring performance and efficiency necessary for long-term optimization. It is also required for long-term robustness and resiliency against environmental, social, and economic shocks. Therefore, sustainability enhances adaptability and adjustability to changes, which are vital for performance improvement in diverse circumstances, another key property needed for antifragility. If a system is resistant to change (e.g., highly dependent on some resources), it may fail if it cannot adapt to new or different situations and evolve at critical points in dynamic conditions. All of these properties support performance that thrives and avoids failure. However, skin in the game, i.e., directly facing negative performances and failures, boosts motivation, reliability, fairness, continuous improvement, and long-term success. It also reduces unnecessary short-term benefits, reckless judgments, and careless analyses and decision-making. We discuss the second layer of attributes/features that are beneficial to one or multiple of these properties in the following section.

3.2 Attributes helping and leading to the necessary properties

The basis of robustness is the correctness and consistency of operations over time, as well as the ability to withstand even sudden changes, which are referred to as *reliability* and *stability*, respectively. A vulnerable system with significant performance variations and dependencies cannot resist notable shocks and changes. In addition, the ability to succeed and survive sustainably over a long period, referred to as *viability*, plays a crucial role in enduring external forces and reducing vulnerability. Viable systems consider potential risks and vulnerabilities to foster fault tolerance and failure mitigation, e.g., with extra backup components (*redundancy*) and/or quick element changes to reduce the negative impacts and failure against stressors (*absorption*). Viability can also assist in resiliency and sustainability.

On the other hand, *optionality* enables various choices, *flexibility* eases configurations, strategy changing, and adjustments, *modularity* reduces dependency by considering interchangeable components and supports adaptability, *agility* empowers quick reaction to changes and swift decision-making, and *non-monotonicity* follows adaptability and strategy changing instead of a unique and linear pathway. All of these features strengthen dynamic adaptability, which is substantial for antifragility. Indeed, adaptability is the main aspect of antifragility. However, the difference between regular robustness and resilience lies in the mindset of using adaptations and adjustments not only to pass through shocks, but also in cases where there are opportunities to grow and improve performance.

Finally, *adding randomness* or controllable shocks and stresses to simulate unpredictability, avoid stagnation, and boost adaptability and performance by *learning from failures* is more efficient when having the skin in the game. Also, general *simplicity* to keep the decision-making process efficient and reduce (unnecessary) complexity conflicts with *flexibility*, emphasizing balancing between the features in a practical anti-fragile system. Of course, *simplicity* also supports robustness and resiliency.

4 How to measure (anti)fragility (RQ1-2)

In this section, we first review and classify how antifragility is quantitatively measured in the literature and how some researchers have utilized the properties and attributes discussed earlier for this purpose. Then, we discuss their limitations and the most promising ways of measuring antifragility.

As shown in Figure 3 all the introduced antifragility measures are based on positive changes in performance as defined by Taleb (Taleb 2012). More generally, one can classify them into three main groups: (1) direct performance deviations, (2) volatility and complexity, and (3) performance improvements of antifragility attributes.

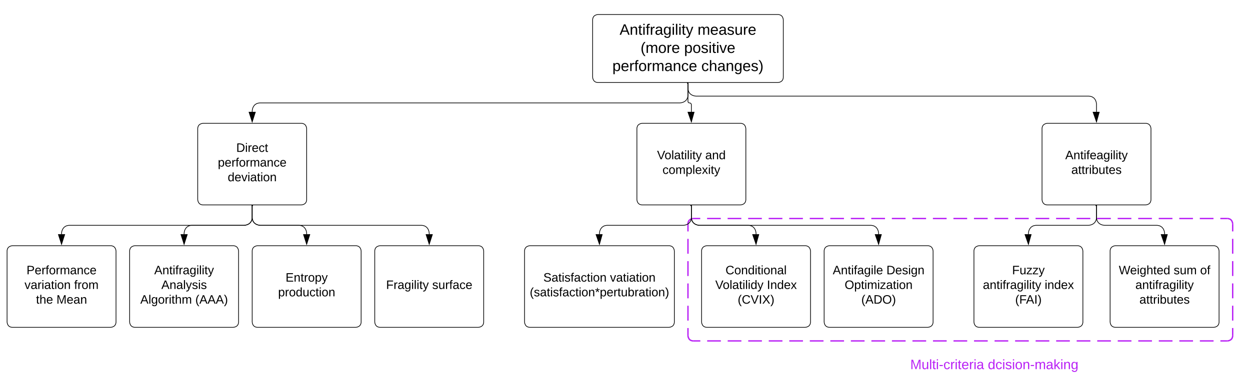

Figure 3: Classification of antifragility measurement in the literature.

The first group, which is based on the main deviation from the performance in shocks, includes: *performance deviation from the Mean*, *antifragility analysis algorithm (AAA)*, *entropy production*, and *fragility surface*. The first two have more solid developments and relative concepts with potential to extend for broader applications, while the last ones have a more particular domain-based focus and/or unclear calculations and assumptions. There are also some other conceptual or problem-specific measures (Kiefer et al.

2018, Bravo & Hernández 2021) that were removed from the classification because they were too subjective or had unclear extendability, mainly related to the first group.

The second group, *satisfaction variation*, *conditional volatility index (CVIX)*, and *antifragile design optimization (ADO)* extended the performance deviation, focusing more on volatility and complexity. For instance, the satisfaction variation approach measures antifragility as the product of satisfaction and perturbation (as a measure of complexity), which may require additional calculations and complexity. Finally, the third group employs multi-criteria decision-making (MCDM) to simultaneously improve and compare the trade-offs between various antifragility properties and/or attributes, thereby finding balanced strategies. This group includes *fuzzy antifragility index (FAI)* and *weighted sum of antifragility attributes*. Note that MCDM has also been utilized in the last two categories of the second group, as highlighted in Figure 3. These measures are reviewed in detail in the remainder of this section before their suitability and pros and cons for optimization-supported decision-making are discussed.

4.1 Direct performance deviations

4.1.1 Performance variation from the Mean

Perhaps the first measurement introduced for (anti)fragility dates back to Taleb's works Taleb et al. (2012), Taleb & Douady (2013), where they defined a heuristic for detecting antifragility as follows:

$$H = \frac{f(\alpha - \Delta) + f(\alpha + \Delta)}{2} - f(\alpha) \tag{1}$$

where $f(\boldsymbol{x})$ represents the performance of the system/model (criteria or objective functions are responses), α is the baseline or the current state of the system/model, Δ denotes a positive or negative deviation from the baseline or the current state. Therefore, equation (1) calculates the (anti)fragility of the system/model as H. Then, based on the definition:

- If $H > 0$, the system is considered anti-fragile, as it benefits from deviations (positive or negative) from the Mean in its state.
- If $H = 0$, the system shows no gain from deviations, indicating robustness but not antifragility.
- If $H < 0$, the system is fragile, as it incurs losses from deviations.

This initial definition and (anti)fragility measurement have been followed by some other authors to calculate and develop new measurements for the antifragility of a system or model.

For instance, de Bruijn et al. (2020) proposed a heuristic measurement to evaluate antifragility in system dynamics. Because the above simple definition can only be applied to two data points, de Bruijn et al. (2020) further validates it under different conditions through sensitivity analysis and Monte Carlo simulations, and is considered a model anti-fragile when the average level of a variable is higher than its original or baseline value. To assess the robustness, recovery, and growth of the model, they manually add small positive and negative shocks to the system through trial and error.

Levin et al. (2014) discussed the results of analyzing the service life extension of aging vehicles and how these results can integrate the concept of antifragility and inform decision-making. Following Taleb's definition (Taleb 2012), to highlight the importance of performance (outcome) convexity for antifragility, they detect fragility when an average performance across scenarios is less than the performance of the average scenario. Indeed, they also considered the Mean performance and the compensation of negative performance with positive ones to demonstrate robustness, while the negative and positive Means describe fragility and antifragility, respectively. They generate various scenarios, including all the combinations of uncertain factors with different values related to different situations, and check whether the performance (or system response) change compared to the baseline (or average scenario) is positive or negative. One of the main findings was that robust strategies can be more fragile than vulnerable ones, which is undesirable in any scenario, highlighting the possibility of robust yet fragile strategies. Therefore, they proposed to consider fragility as a binary value (true/false) rather than a multi-categorical and ordinal measurement.

Besides, Aven (2015) also connects (anti)fragility to vulnerability, noting that low vulnerability leads to robustness. However, they emphasize the consideration of extreme vulnerability scenarios with negative or positive performance changes (which can represent unexpected outcomes or known surprises) in antifragility analysis to explore possible thriving options, perhaps through additional adjustments and adaptations. However, their work is conceptual, with connections to risk analysis and conditional performance in various situations.

Mahata et al. (2021) define financial antifragility of a company i as liquidity-to-expense ratio:

$$\phi_i = \frac{\text{current assets} - \text{current liabilities/debts}}{\text{expenses/cost}} = \text{liquidity balance of the company/expenses}$$

They develop a model to simulate stock price movement during the COVID-19 shock and subsequent recovery as a function of normalized net fund flow due to institutional investors and the stock's antifragility parameter. They considered financial antifragility to be a company's ability to survive and thrive after a recovery period following a financial crisis. It primarily relies on its financial liquidity position to mitigate liabilities. They also exhibited V-shaped and L-shaped recoveries, respectively, for companies with positive and negative antifragility indices. Similarly, as with other measurements, the higher $\phi > 0$, the better the performance. Although their measurement is particularly designed for finance, they followed the general concept of getting better performance after shocks, as defined for antifragility by Taleb (Taleb 2012).

4.1.2 Antifragility analysis algorithm (AAA)

Li et al. (2024) introduce a method, called Antifragility Analysis Algorithm (AAA), to maximize the performance. They extend Taleb's definition of antifragility and consider a decision or strategy anti-fragile if the average performance (or outcome), after adjustments, is significantly better than the average baseline or current performance. Indeed, they measure the strategy's antifragility score as the average change of performance based on the current scenario. Then, they use the experts' verbal judgment matrix to evaluate the strategies' performance in different scenarios. Finally, they calculate the antifragility score for each strategy to rank them based on this score. Their process employs some concepts and steps similar to those utilized in the robust decision-making (RDM) framework (Lempert et al. 2006) to conceptualize AAA in decision-making. However, they limited their method to using a simple scenario planning approach and a verbal judgment matrix that was limited to a few strategies and scenarios.

4.1.3 Entropy production

Following the definition of Taleb (2012), López-Corona & Kolb (2021) approximate the system stability or antifragility measurement as a payoff time series (response) utilizing the Fisher information for planetary antifragility. However, it is more conventional and is based on the entropy production of ecosystems available from existing databases.

4.1.4 Fragility curve/surface (failure probabilities)

Niemeier (2022) considers the probability of production failure (the probability of failure in limits) in a reliability analysis to decrease the required number of simulations in an optimization problem. Then, they calculate a fragility curve/surface to see where a failure might happen based on some probabilities (coming from the simulations in those areas) in a reliability analysis. The explanations are limited and can be found in only a few short conference papers and reports. Additionally, it relies entirely on known information related to failure probabilities, which is often impossible to calculate in advance.

4.2 Volatility and complexity

4.2.1 Satisfaction variation

Based on the variation around the shock or perturbations in the decision-maker's satisfaction, Pineda et al. (2019) propose a pragmatic measurement for the antifragility of complex systems. Indeed, they define antifragility as the product of *perturbation* and *satisfaction* between 0 and 1, where a system is robust when the antifragility score is close to zero and fragile when negative values (to -1) are observed.

$$Antifragility = -\Delta\sigma \; \Delta x \tag{2}$$

where denotes the degree of perturbation, and the difference of satisfaction around the perturbation is described by $\Delta\sigma = (C - C_0) \in [-1, 1]$, C and C_0 are the complexity after and before the perturbations.

This measurement has been tested in the context of random and biological Boolean networks (Pineda et al. 2019), multi-layer random Boolean networks (Kim et al. 2019), convolutional neural networks (Kim et al. 2020), and ecosystems (Equihua et al. 2020). Besides, Alatorre et al. (2023) have also followed this definition and considered a similar antifragility measure as the product of *perturbation* and *satisfaction* in the cryptocurrency market. They measure satisfaction as the *price returns* and the perturbation as the mean of different volatility parameters, including the *volatility index* (VIX) at a time.

4.2.2 Conditional correlation with Volatility Index (CVIX)

Gotardelo & Goliatt (2024) consider CVIX (Conditional Correlation with VIX), as an anti-fragile metric, to account for market volatility in the portfolio selection problem and check if they can find a better portfolio return using this model compared to the traditional CAPM (Mean-Variance) model. In this sense, they show that having conditional correlation to VIX (in a way that increases in volatility) slightly declines assets, and drops in volatility (leads to a price rise in assets) are desirable. Indeed, if $CVIX^{+}$ ($CVIX^{-}$) is the correlation coefficient of the index, for peak (trough) positive (negative) VIX, CVIX can be calculated as follows:

$$CVIX = CVIX^{-} - CVIX^{+}$$

They optimized portfolios to minimize CVIX (i.e., antifragility–better performances in increased volatility), using a multi-objective optimization methodology involving convex (mean, variance, skewness and kurtosis) and non-convex (Omega, maximum drawdown, CVIX) attributes.

Then, they compare four models: (1) CAPM; (2) Omega CAPM; (3) with only the convex attributes (Kurtosis, Mean, variance, and expected loss); and (4) the one augmented with nonconvex attributes (Omega, skewness, kurtosis, Mean, CVIX, and drawdown). The first two were single-objective, while the latter two were multi-objective models. However, the third model employed the weighted sum method to aggregate the objectives, and therefore, only one Pareto optimal solution was identified. Finally, they achieved better results in portfolio return by using the last model, which included the CVIX index, and concluded that considering nonconvex attributes can enhance antifragility and result in better performance when facing crises compared to mean-variance models. Indeed, avoiding too sensitive assets with extreme negative performance facing the volatility can improve the system's performance and its average.

4.2.3 Antifragile Design Optimization framework (ADO)

Coppitters & Contino (2023) propose a multi-objective approach, called the Antifragile Design Optimization framework (ADO), to optimize the central tendency, variability, and antifragility of renewable energy systems under uncertainty. They consider three objectives: (1) the performance in central tendency of the distribution (expected performance); (2) variability (Upside Potential Ratio (UPR), which protects against downside variation while promoting upside variability); and (3) antifragility (skewness to represent antifragility in the quantity of interest). Indeed, skewness, considered as the measurement of antifragility, calculates the probability of negative or positive performance (the difference from the minimum acceptable performance level) across all scenarios. Therefore, positive skewness (skewed to the right) indicates a higher probability of achieving a positive performance in most scenarios (reaching better performance than the minimum acceptable level in more than half of the scenarios), suggesting antifragility. Fragility can also be identified by negative skewness. They consider three extremes from the Pareto optimal designs, calculated via the NSGA-III algorithm (Jain & Deb 2013), and compare their robustness (worst-case) in reaching them. Their results showed that the optimized skewness is skewed towards positive performance, and thus, is referred to as an antifragile design/response.

4.3 Multiple antifragility attributes

4.3.1 Fuzzy antifragility index (FAI)

Ghasemi & Alizadeh (2017) use questionnaires as a qualitative method to elicit expert's opinions on seven antifragility features (Absorption, redundancy, Stress Starvation, Non-monotonicity, Requisite variety, Emergence, Uncoupling) gathered from the literature, and Sharon's entropy to find fuzzy weights (lower, median, upper) for multicriteria analysis and ranking of these features. Then, they calculate a fuzzy antifragility index (FAI) using the average fuzzy weights and the average fuzzy ratings (R_j) as follows:

$$FAI = \frac{\sum_j w_j R_j}{\sum_j w_j}$$

Finally, they compute the Euclidean distance between FAI and the linguistic term in the case study.

4.3.2 Weighted sum of antifragility features

Moudi et al. (2020) propose a fragility index for managing water supply systems. As the only way of measuring/considering (anti)fragility in the system's optimization models, they introduced a three-objective optimization model to minimize the fragility of a water supply system, which includes simultaneous minimization of not being *reliable*, not being *resilient*, *vulnerability* of the system. These (anti)fragility attributes or objectives have been defined as: **reliability:** the probability of meeting the demand; **resiliency:** the probability of not seeing any shortage issue in a period; and **vulnerability:** the severity of a failure, e.g., the magnitude of water scarcity associated with this system failure (i.e., the ratio of shortage in supply over the demand).

They consider the weighted sum of these three attributes as a *fragility index* to be minimized and examine two other future scenarios besides the current state (the average annual precipitation): *slight* and *extreme drought*, based on 10% and 30% reductions from the average historical rainfall-related data. They also generate different Pareto optimal solutions by changing the weights of various objective functions, where the fragility scores increase when the reliability weight increases. However, as their problem was a mixed-integer linear problem, it is possible that a weighted sum method could not produce a good representative set of Pareto optimal solutions, particularly in the middle and possibly nonconvex (or gap) regions (Miettinen 1999).

4.4 Limitations of the existing measurements

After reviewing and classifying the existing measurements developed for antifragility in the literature, we discuss their limitations and important aspects for measuring antifragility in optimization-supported decision-making. Table 3 summarizes the limitations of the existing measurements described in the previous section.

As can be seen in this table, many measurement approaches *focus too much on average performance* (Taleb & Douady 2013, de Bruijn et al. 2020, Levin et al. 2014, Coppitters & Contino 2023) and ignore compensation of a few significant negative performances by many small positive ones, which is conceptually in contrast to antifragility. For instance, it is possible that a negative loss of €10 billion in one scenario is compensated by 10×€1 billion in positive performances in 10 (or more) other scenarios or times. However, instead of being hidden and ignored, this situation can be viewed as a fragility of the system, motivating decision-makers to find adaptations that enable them to avoid loss and/or thrive in that particular scenario. A similar situation can happen when multiple (different) shocks or crises can affect the performance of a system.

Measure category		Limitations
Direct performance deviation	Performance deviation from the Mean	- Focusing on average performance - Emphasizes infinite negative performance - Ignore compensation of many or significant negatives compensated by many small positive performances - Might only consider (anti)fragility categorically (true/false)
	Antifragility analysis algorithm (AAA)	- Only can handle a limited number of strategies and scenarios - Required high cognitive load for judgment and comparisons in many scenarios - Depends on expert judegement
	Entropy production	- Too complex and requirement of case-specific definition and formulation for complexity/entropy
	Fragility surface	- Depends on the knowledge of the probability of failures - Extremely limited description for the generalization or even the same usage
Volatility and complexity	Satisfaction variation	- Too complex - Based on unclear consideration of complexity and satisfaction - Needs specific adjustment and formulation in different cases
	Conditional volatility index (CVIX)	- Too complex and requirement of case specific adjustments - Only compared with strategies that were generated without volatility consideration
	Antifragile design optimization (ADO)	- Focusing on average performance - Required probabilistic framework - Depends on initial input distributions - Ignore compensation of many or significant negatives compensated by many small positive performances
Antifragility attributes	Fuzzy antifragility index (FAI)	- Only can handle a few strategies - Depends on unclear rating and fuzzy weights
	Weighted sum of antifragility attributes	- Issues of aggregating multiple criteria - No specific measure

Table 3: Limitations of the existing antifragility measures.

Besides, many measurements are *too complex* (Pineda et al. 2019, Kim et al. 2019, 2020, Equihua et al. 2020, Alatorre et al. 2023) and/or *need specific design and reformulation for different application domains* (López-Corona & Kolb 2021, Mahata et al. 2021, Niemeier 2022, Gotardelo & Goliatt 2024), which limit the possibility of re-usage and generalization. Additionally, there are measurements that have a specific design and methodology for evaluating only a few strategies in a limited number of possibilities or future scenarios (Ghasemi & Alizadeh 2017, Li et al. 2024). Finally, some others require knowledge of probabilities, distributions, and their calculations (Niemeier 2022, Coppitters & Contino 2023), which is again contrary to the philosophy of considering rare events and Black Swans for deep uncertainty and anti-fragile decision-making.

Therefore, a proper measurement for anti-fragile decision-making should be designed in a way that (1) is simple enough to be used, generalized, and understood by various involved parties in decision-making in different application domains, (2) provides a way for adaptation identification, not only for the events or situations with large negative consequences but also when reaching large positive benefits are possible, (3) not heavily rely on the exact knowledge of probabilities and predictions, and (4) be able to be utilized in problems with many strategies and scenarios, such as multi-criteria optimization-supported decision-making under deep uncertainty. The next section proposes a proper measurement for antifragility and explores potential applications of it in informed decision-making.

5 Proposed antifragility measurement and possible ways of its usage in anti-fragile decision-making (RQ2)

5.1 Proposed antifragility measurement

Based on the above discussions, we propose to first set a threshold (or tipping point) for antifragility in each criterion; then, if the related performance/outcome passes that threshold in a scenario, there is a fragility in that scenario. Otherwise, it is anti-fragile based on the given criterion in that scenario. Therefore, after considering all scenarios, *the number (or portion) of scenarios in which a strategy j fails with respect to a criterion (or objective) i, called a fragility score and shown by F_{ij}*. Then, the antifragility score for a strategy/decision j in criterion i (AF_{ij}) can be measured as

$$AF_{ij} = 1 - \frac{F_{ij}}{\text{number of scenarios}} \tag{3}$$

Therefore, a strategy is 100% anti-fragile based on a criterion if the antifragility threshold has not been passed in any scenario. Being 100% anti-fragile based on all criteria reflects the overall antifragility. The antifragility score is between 0 and 1 for each criterion, and the overall or total score can be considered as the average antifragility in all criteria for each strategy j:

$$TAF_j = \frac{\sum_{i=1}^{n} AF_{ij}}{n} \tag{4}$$

However, an initial separate investigation of fragilities in different scenarios is highly recommended to avoid the issue of hiding some results in aggregation metrics. In general, as the default option, the antifragility threshold can also be the average performance or less than the performance in the baseline scenario. Otherwise, decision-makers can directly set thresholds if desired. Similar to robustness (e.g., Shavazipour et al. (2021a), Shavazipour, Eyvindson & Podkopaev (2025)), having trade-offs for antifragility between different criteria and/or scenarios are expected, and similar visualization tools (e.g., (Shavazipour et al. 2021b)) should be considered for comparison and decision-making.

The proposed antifragility measurement satisfies all the recommended properties mentioned in Section 4.4, as the proposed measurement: (1) is simple for understanding and calculations, (2) eases the comparison of augmenting the initial/previous and possible adaptation strategies in various scenarios, (3) does not require any probability information, and (4) can easily integrated in strategy evaluations and decision-making processes. Besides, the proposed measurement can also be considered as an objective function to be maximized in an optimization model. Therefore, feasible strategies with (low) antifragility scores can be removed easily.

5.2 Antifragility measurement usage in decision-making

As previously discussed, anti-fragile systems and related decisions require having more potential for performance improvements in some scenarios, while also tolerating small negative performance drops in other scenarios, highlighting the importance of performance (outcome) convexity for antifragility. However, most existing approaches focus only on extreme situations, mainly because Taleb's intention and definition targeted these extremes. Still, many other similar situations should be covered, such as a significant negative performance in one scenario versus many smallish positive performances in many other scenarios (matches the robustness definition). Such a consideration can easily turn the problem into an anti-fragile one, allowing for the development of adaptation strategies to achieve even greater positive performance in that scenario. Also, it is reasonable to evaluate the situation of compensating for negative performances in response to one shock with positive performance in response to another shock(s) in the overall performance. Besides, the antifragility in transition to adaptations and contingency plans for various scenarios must also be evaluated to avoid introducing new fragilities.

These essential aspects of adjustment and their turning point must be taken into consideration in the decision-making process. Therefore, we propose considering possible ways for utilizing antifragility measurement in anti-fragile decision-making. Since decision-makers must simultaneously consider multiple aspects and attitudes for antifragility, our proposals incorporate multi-criteria decision-making. We first discuss some possible ways of anti-fragile analysis and decision-making, before proposing a generic modular architecture as the most promising approach for anti-fragile decision-making, to tHink beYonD Robustness And resilience (HYDRA), in the next section.

MCDM can be divided into two territories: (1) generating alternative strategies/decisions, and (2) analyzing these alternatives based on multiple criteria, mainly to evaluate the goodness or performance of these alternatives from various aspects. Accordingly, we discuss potential ways for each territory.

5.2.1 Anti-fragile strategy generation

Here, we assume that alternative strategies or decisions are not already known or separately generated. Therefore, we first need to identify or generate strategies or decisions that not only work in various future conditions, withstand, and/or recover from shocks, but also include possibilities for growth in some scenarios facing changes and shocks. Hence, relevant antifragility criteria or objectives (aspects, properties, and/or attributes) must be considered and satisfied. In non-model-based approaches, such strategies can be determined through experts' opinion in specific workshops and quality methods, e.g. Derbyshire & Wright (2014).

However, as the focus of this study is on model-assistance and particularly optimization-based decision-making, our recommendation, as one way of strategy generation using optimization-based models, is to consider antifragility criteria as additional objectives and combine them with problem objectives, as an extended problem, for a multi-criteria decision analysis or multi-objective optimization model. If the number of criteria are too many making challenges in solving the problem and increase the complexity in comparison and decision-making, the antifragility criteria (or some of them) can also be aggregated, e.g. as a weighted sum (e.g., similar to Moudi et al. (2020)) and/or relocated some of them as constraint with bound thresholds, e.g. similar to $\epsilon-$constraint method (Haimes 1971). However, as mentioned earlier, the issue of blind usage of aggregation must not be ignored without separate investigations.

It is also possible to consider an antifragility measurement (e.g., the proposed measure in equation (3) or (4) in 5.1) as an additional objective to be maximized or conduct a separate antifragility analysis, e.g., similar to Coppitters & Contino (2023). Nonetheless, due to existing conflicts between some criteria, a similar aggregation can conceal some of these conflicts, therefore necessitating a separate consideration of them or groups of them. Such a grouping can be achieved through a similar classification of properties described in Section 3.

Also, note that simply adding antifragility criteria may not necessarily transform an existing multi-objective optimization model into one that produces anti-fragile strategies, although it can facilitate antifragile decision-making and identify more anti-fragile strategies. Indeed, identifying vulnerabilities/fragilities under particular conditions and generating suitable adaptations and contingency plans require a broader framework integrating different approaches. We will propose such a framework in the next section.

5.2.2 Anti-fragile analysis and decision-making of known alternative strategies

If alternative strategies are known (generated either by experts or optimization models), a multi-scenario multi-objective optimization (MsMOO) model can be formulated to investigate and compare the trade-offs between different aspects and/or attributes of antifragility. Of course, it can also be incorporated into an optimization-supported interactive framework such as Shavazipour, Kwakkel & Miettinen (2025). For this purpose, based on the antifragility definition and the proposed classification in Section 3, an MsMOO model can be formulated in three ways: **A. a performance-based model**, **B. a model based on antifragility properties and attributes**, **C. a combination of them**. Here, we discuss the initial details of each possible model; however, further discussion, validation, and comparison may require a separate study, which we reserve for the future.

A. Performance-based model

As discussed earlier, the definition of antifragility is based on deviations from the average performance or differences from the baseline scenario. We also warned of the possibility of compensating for a large negative performance in a scenario with many small positive ones, highlighting the importance of considering both negative and positive performances to avoid being misled by aggregations. Accordingly, the following criteria can simultaneously be considered in a performance-based multi-scenario multi-objective optimization model incorporating these factors for antifragility analysis and comparisons:

- **Maximizing positive deviations from the baseline performance:** Different options, such as total aggregation of differences or max-min differences in all or some selected scenarios, can be considered.
- **Minimizing negative deviations:** Similarly, one can consider the total aggregation or min-max differences from the baseline scenario in all or some selected scenarios.

Besides, constraints can also be added to avoid (extremely) large negative performance in each scenario to ensure the consideration of large negative deviations.

B. Attribute-based model

Another way is to consider multiple antifragility properties and/or attributes, described in Section 3 and formulate an MsMOO model incorporating multiple antifragility properties. Similar to strategy generation, an aggregation of certain groups of properties and/or attributes can be considered to reduce the number of criteria or objectives, if desired.

C. Mix performance-attribute-based model

Additionally, performance- and attribute-based objectives can be combined in a single MsMOO model, or some of them can be considered during strategy generation. Besides, an antifragility measurement, e.g. the proposed one in equation (3) or (4), can also be combined or considered as an extra criterion/objective in the MsMOO model, or in the relevant strategy generation step. The number of criteria or objectives and how to consider them in strategy generation and/or antifragility analysis depend on the specific problem and the decision-makers involved.

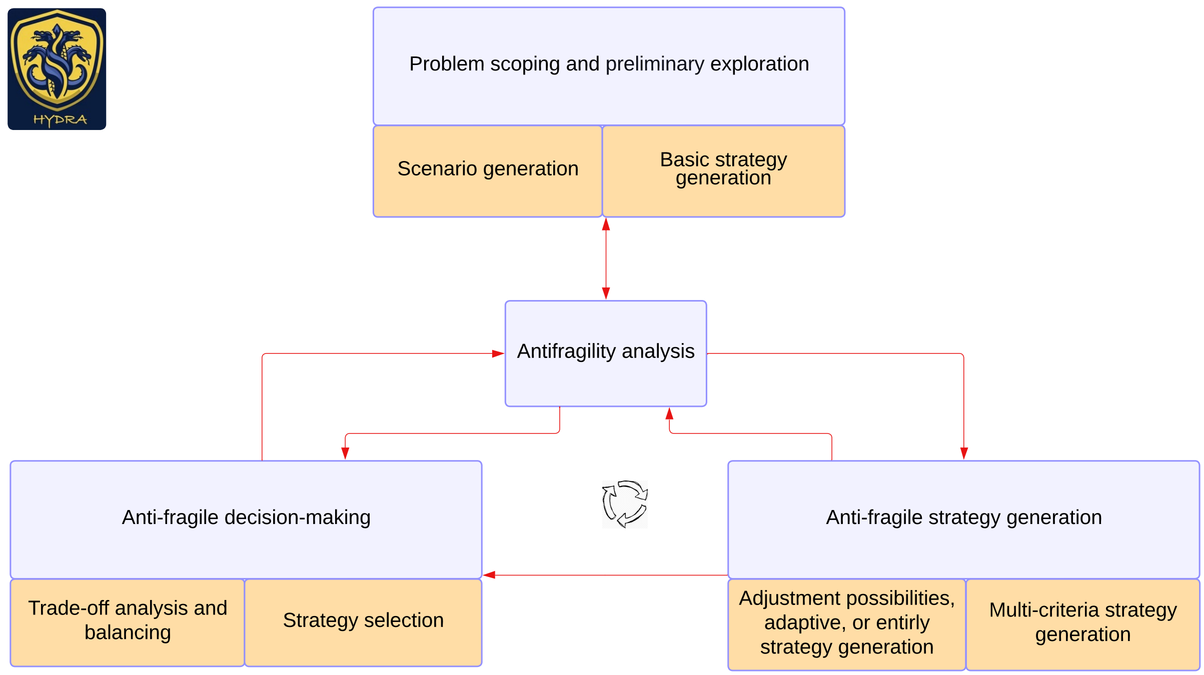

Figure 4: A generic scheme of the proposed anti-fragile decision-making framework (HYDRA).

6 Proposed anti-fragile decision-making framework, HYDRA, (RQ3)

Apart from measuring antifragility and strategy evaluations, supporting decision-making in our complex, deeply uncertain world, and managing anti-fragile planning is crucial. Therefore, in this section, we propose a novel generic framework and modular architecture for antifragile thinking and decision-making, called HYDRA. The HYDRA framework can be utilized in both non-model and model-based (optimization-supported) decision-making and includes four iterative main steps: (1) *Problem scoping and preliminary exploration*, (2) *Antifragility analysis*, (3) *Anti-fragile strategy generation*, and (4) *Anti-fragile decision-making*.

The general scheme of the proposed HYDRA framework, including its four main iterative steps and their lower-level sub-steps, is described in Figure 4. All these steps are to be iterated as many times as required to confirm the formulated decision problem, including testing and validation, antifragility analysis, strategy generation, and decision-making. The details of the steps are described below:

Step 1. Problem scoping and preliminary exploration

The first step of this iterative and interactive HYDRA framework is to frame the decision-making problem and ensure that, for example, all its characteristics, requirements, decisions to be made (decision variables), evaluation criteria, performance measures (objective functions), limitations (constraints), possible impacts, and both certain and uncertain factors/parameters are identified. While the decision-making problem and its details are often demonstrated, mainly qualitatively in non-model-based approaches, model-based approaches use the information to formulate a model for quantitative analysis representing relationships, evaluations/performances, and outcomes, e.g. an optimization model. Note that both analytical and non-analytical models (e.g., surrogates) can be considered and/or combined.

Step 1a. Scenario generation

Based on the existing uncertainty in the decision-making problem, scenarios can be built to describe various states of the world. Depending on the problem characteristics and the level of uncertainty (Walker et al. 2013, Shavazipour & Stewart 2021), various scenarios can be generated, for example, using expert judgments to generate a few scenarios similar to scenario planning (Van der Heijden 1996), or utilizing different parameter values to develop hundreds/thousands scenarios as conducted in decision-making under deep uncertainty methods (Eker & Kwakkel 2018, Marchau et al. 2019, Shavazipour et al. 2020, Shavazipour & Sundström 2024).

Step 1b. Basic strategy generation (mainly focusing on a single criterion)

As the final step of this phase, before any analysis, we need to identify some possible strategies or decisions for the decision-making problem. Similar to the previous step, experts (or decision-makers) can identify these strategies or be involved in relevant model development and validation, based on the problem. The aim of this initial step is not to identify advanced or perfect strategies that work in all conditions but only to generate some feasible ones, e.g. for specific scenarios, to start with. For instance, experts can separately focus on a scenario and provide feasible strategies. Alternatively, single-scenario, single-objective optimization models can be formulated and solved to generate optimal strategies for specific scenarios separately (see, e.g., generating ideal values for objectives in different scenarios in Shavazipour et al. (2021a)).

Step 2. Antifragility analysis

Use the proposed measurement in 5.1 and/or formulate an MSMOO model in one of the ways proposed in Section 5.2.2 to analyze the (anti)fragility of the generated strategies in various conditions and scenarios. Then, possible adaptation and contingency strategies can be investigated, mainly for fragilities and vulnerable scenarios, as well as other opportunities for performance improvement and thriving.

Indeed, if there is no fragility or vulnerable scenario that can be seen as a consequence of applying a strategy, it is considered an anti-fragile strategy. If some fragility exists, the strategy is considered fragile or anti-fragile to some extent. Then, these (anti)fragility scores and possible trade-offs can also be compared as explained in Section 5.1.

Step 3. Anti-fragile strategy identification

After identifying the fragilities, possible adaptation strategies and relevant models can be considered. Any fragility may require different (extra or specific) transformations and/or models, such as for multi-hazard problems. Therefore, multiple ways of adaptation and model developments to various fragilities in particular scenarios are expected. Also, a totally different and new strategy, e.g., a new technology, can be considered affecting the change and appendices to the whole problem and in all scenarios. Indeed, the vital step is to identify possible adaptation strategies in fragile scenarios (or when facing shocks) based on experts' opinions and/or various models—i.e., the possibility of converting threats into growth opportunities.

Of course, the possibility of such changes and adaptations can also be considered in non-fragile scenarios if desired. Note that seeking opportunities for significant growth under regular conditions, when performance is satisfactory, is uncommon and may be overlooked in favor of addressing fragilities. However, if time allows and it's desired, such an investment can literally help improve overall performance and even enhance the system's antifragilities.

This step can be divided into two smaller steps: (a) finding overall possible strategy changes for performance improvement and new potential opportunities, and (b) advanced specific adaptation strategy generation for such scenarios in combination with the initial strategies or as a totally new strategy. Remember that each of these steps (and their sub-steps) can be repeated multiple times interactively with the experts, decision-makers, and/or stakeholders. It is also possible to return to any previous step at any time if required.

Step 3a. Adjustment possibilities and adaptive strategy generation

Before developing any new or adaptation strategy, or a relevant model for improvement, one needs to check if such a positive change is possible and makes sense because no adaptation or new strategy might be possible (or at least easy or meaningful) in some conditions. For example, if a large meteor survives its passage through the Earth's atmosphere and unexpectedly heats the Earth, destroying everything, no adaptation or consistency decision may be possible. Therefore, all possible adaptation strategies, including additional or entirely new strategies, for addressing various fragilities and opportunities must be identified first before determining the strategies.

Step 3b. Multi-criteria strategy generation

As discussed earlier, due to the multi-criteria nature of the problems and the sustainability concerns of various involved parties and stakeholders, strategies should be generated and evaluated from different aspects and criteria. Again, the strategies (new strategies or meta-strategies, initial or first-stage strategies and relevant possible adaptations) can be generated by experts, models, or both as suggested in Section 5.2.1. As discussed, the antifragility of possible strategies directly identified by experts can be investigated using one of the MsMOO models described earlier (in Section 5.2.2) or the proposed measurement in the antifragility step. Alternatively, experts can support model development for generating anti-fragile strategies and their evaluation and validation, as demonstrated in Section 5.2.2 before the antifragility analysis. For instance, the multi-stage multi-scenario multi-objective robust optimization framework (Shavazipour & Stewart 2026) can be used to construct and adjust to simultaneously generate initial and possible adaptation strategies. Adding an extra objective, such as antifragility, can enhance the identification of anti-fragile strategies. To add completely different and new strategies, novel models can also be formulated. In any case, as suggested in Shavazipour & Stewart (2026), continuous monitoring and the development of moving horizon models can also support dynamic adaptations in anti-fragile decision-making (AFDM).

Step 4. Anti-fragile decision-making

After generating anti-fragile (meta-)strategies and conducting initial investigations in their antifragility analysis, a more in-depth trade-off analysis, balancing, and strategy selection are required. Indeed, due to various existing trade-offs (e.g., between objectives, scenarios, and their antifragility), which make the decision-making problem high-dimensional and complex, multiple iterative analyses and selections are required.

Step 4a. Trade-off analysis and balancing

First, alternative (meta-)strategies must be compared based on multiple evaluation criteria and their various consequences in different scenarios.

Step 4b. Strategy selection

If the number of alternative strategies is high, e.g. when optimization-supported models are utilized, a manageable number of (meta-)strategies should be selected initially to reduce the complexity, computation expenses, and the cognitive load—perhaps after some trade-off and/or antifragility analyses. Then, another comparison and analysis can be done with fewer alternatives in various iterations until the most preferred (meta-)strategy is selected as the final one to be implemented, e.g., in a similar way proposed in Shavazipour, Kwakkel & Miettinen (2025).

Usually, these steps should be done interactively multiple times and perhaps go back to some other steps to select the final (meta-)strategy. To keep the proposed HYDRA framework generic and to account for the possible requirements of particular problems, problem-specific adjustments are left for the application of HYDRA in different decision-making problems in the particular fields.

	Wheat			Corn			Soybeans		
	s_1	s_2	s_3	s_1	s_2	s_3	s_1	s_2	s_3
Yield (Tonnes/hectare)	4	3.6	3.3	9	6	5	2.2	2	1.5
Planting Cost (€/hectare)	240	250	275	430	450	475	290	340	390
Selling Price (€/tonnes)	160	185	210	125	145	165	305	330	350
Environmental benefit coefficient (percent)	14	13	12	12	13	14	22	24	25
Purchase Price (€/tonnes)	180	200	230	140	160	180	N/A	N/A	N/A
Minimum Requirement (tonnes)	400	400	400	440	440	440	N/A	N/A	N/A

Table 4: Farming data set, adapted from Shavazipour & Stewart (2021).

7 An illustrative farming example

Here, we consider a simple farming example (Shavazipour & Stewart 2021) to showcase the utilization of the proposed HYDRA framework and antifragility measurement.

`Step 1. problem scoping and preliminary exploration`
Then, consider a farming problem where a 1000 hectare land is regularly used to grow various products such as wheat, corn, or soybeans. Also, there is a cattle that requires 400 tonnes of wheat and 440 tonnes of corn to be fed. The farmers usually feed the cattle with their own production of wheat and corn, but in case of a shortfall, they must be bought from a wholesaler at a cost of €200/tonne of wheat and €160/tonne of corn. However, extra production can be sold for €185/tonne of wheat and €145/tonne of corn. Besides, soybeans can also be grown, which can be sold for € 330/tonne.

To consider uncertain weather conditions, e.g., rainfall and heat, affecting the crops' yields, three scenarios of (1) exemption (s_1), average (s_2), and poor (s_3) weather conditions are evaluated. Table 4 demonstrates different values for uncertain parameters under different scenarios (`Step 1a`). Organic farming produces environmental benefits, resulting in some tax reduction calculated from an environmental index of each product shown in Table 4. The planting costs for each product, along with the selling and purchasing prices, and minimum feed requirements, for different scenarios, are also portrayed in the table.

This two-stage decision-making problem includes deciding the portion of land to be allocated for planting each product ($x_i^0, i = 1, 2, 3$) at the initial stage without knowing the exact weather scenario, and possible decisions of selling and/or purchasing crops from the wholesaler in the second stage when the weather scenario is realized ($x_{ik}^1, i = 1, 2, 3$ (selling tonnes of each crop) and x_{4k}^1, and x_{5k}^1 (purchasing tonnes of wheat and corn, respectively).

Then, considering the three objectives of f_1 : minimizing the costs, f_2 : maximizing income, and f_3 : maximizing environmental benefits, one can formulate a two-stage multi-objective optimization model as described in Shavazipour & Stewart (2021, equation (5)). We can identify dynamic robust Pareto optimization by solving this problem (`Step 1b`). Figure 5 compares the objective function values of sixteen Pareto optimal solutions to this farming example (taken from Shavazipour & Stewart (2021)) across all three scenarios, highlighting possible ranges of each objective function for these strategies across all scenarios.

`Step 2. Antifragility analysis`
Now we utilize the measurement proposed in Section 4 to analyze the fragility scores of these strategies. Therefore, we first set the fragility thresholds, as *cost* $>$ € $400,000$, *income* $<$ € $400,000$, and *environmental benefits* > 150 units. Note that considering equal thresholds outcomes for the first two objectives, cost and income, represents the break-even point as the economic fragility of the business, and the third score is the average outcome (performance) of all the solutions for the corresponding objective functions matching the original definition of antifragility.

Then, for each solution in each objective, we check how many scenarios we could satisfy these thresholds. Table 6 demonstrates the number of scenarios in which the corresponding strategy is fragile. As can be seen

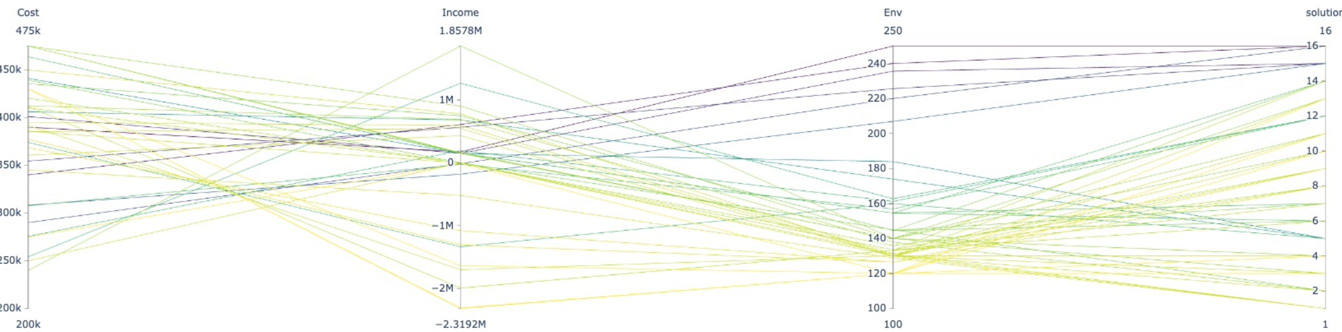

Figure 5: Sixteen Pareto optimal strategies to the two-stage farming problem under three scenarios.

in this table, none of the strategies is antifragile in all three scenarios. Particularly, none of the solutions are antifragile in all scenarios based on the second objective, income, highlighting possible fragility by not meeting the break-even point. The best total antifragility scores (TAF_{ij}) recorded for strategies '12' and '16' are 0.77, corresponding to their antifragility based on the first and the third criteria, while both are fragile in two out of three scenarios in the second criterion (income). It is possible to generate all (or as many as possible) solutions using the model, but similar fragility scores for some scenarios can be expected [1]. In contrast, exploring the possibility of different strategies that are anti-fragile in all scenarios or adaptive strategies to decrease the fragility in some scenarios through scenario-specific adaptive approaches, or an entirely novel solution or model may be beneficial.

Criterion	Strategies															
	1	2	3	4	5	6	7	8	9	10	11	**12**	13	14	15	**16**
f_1	0	2	0	3	2	3	3	1	1	3	3	**0**	3	3	1	**0**
f_2	2	2	2	2	2	2	2	2	2	2	2	2	1	2	2	2
f_3	3	3	3	3	0	2	2	3	3	3	3	**0**	3	2	0	**0**
TAF_j	0.44	0.22	0.44	0.11	0.55	0.22	0.22	0.33	0.33	0.11	0.11	**0.77**	0.22	0.22	0.66	**0.77**

Table 5: The fragility and antifragility scores of 16 solutions across three scenarios.

`Step 3. Anti-fragile strategy generation`

It already showed in Shavazipour & Stewart (2021) that dynamic robust decisions outperform traditional robust decisions. It is also possible to generate optimal solutions for each objective in each scenario condition. However, as seen earlier, those strategies might yet be fragile in some scenarios. Then, the question is how anti-fragile decisions can be made?

As discussed earlier, after identifying some fragilities in the system/process, the problem must be investigated further to identify the possibility of any adaptation strategy or a totally different innovative strategy to reach antifragility. Here, in the farming problem, we consider testing a new *agrivoltaic* strategy, which considers building a solar farm in combination with traditional agriculture for simultaneous crop and electricity production, showing beneficial potentials (Dinesh & Pearce 2016). Besides electricity production, *agrivoltaic* systems are particularly suitable for warm and dry areas with water scarcity and significant heat, e.g., for utilizing shading effects and growing crops under solar panels (Semeraro et al. 2024, Williams et al. 2025).

For simplicity, we assume no new scenario will arise and consider the same three scenarios as before. Additionally, we assume similar benefits from shading for all three crops, although corn may receive a slightly lesser benefit than the others. In reality, the angle of panels can also be controlled, and/or the

[1] Note that the purpose of this example is to showcase the application of the proposed antifragility measurement and framework, and not to identify the best suitable solution

generated electricity can be used to buy extra water for irrigation of the plant if required. However, we have not considered these details to keep the simplicity in showcasing the proposed framework. Also, note that all the data utilized here are slightly hypothetical for illustration purposes.

Therefore, in the new model, a new decision variable x_4^0 is introduced to allocate each hectare of land for installing an agrivoltaic system, which covers 60% of the land for this system, including the required space for farming machinery, and the remaining 40% for agriculture. So, the total available land will be reduced if any part of the land is allocated for an agrivoltaic system. To ensure this, we incorporate the new decision variable into the land capacity constraint. Since farming is still possible on the remaining 40% of the lands allocated to the agrivoltaic system, another extra set of decision variables y_1^0, y_2^0, y_3^0 is also considered for deciding the partitions of that to each crop, so the aggregation must be equal to the land allocated to the agrivoltaic system (x_4^0). We also assume that all the electricity produced can be sold at the same selling price. The cost of installing such a system in each hectare is $400,000$ with an average lifespan of 25 years. Therefore, the annual cost, including maintenance and insurance, is approximately €$17,000$. We also consider the average price of selling a kW of electricity as 0.1 and the electricity capacity of, respectively, 1 000 000, 800 000, and 600 000 kW electricity production by each hectare allocated to the agrivoltaic system in each scenario.

Besides, producing solar electricity significantly increases various environmental benefits, including energy production, carbon emissions reduction, land-use efficiency, crop yield increases, water conservation, soil benefits, and biodiversity support. Here, we consider the effects on the yield production and electricity production on the overall environmental index. Finally, the yield change for wheat are -5%, -10%, and $+10\%$ in scenario s_1, s_2, and s_3, respectively. Corn's yield change are -10%, -15%, and $+5\%$ in each scenario, and the changes in soybeans are -5%, -10%, and $+15\%$. The formulation of the new two-stage multiobjective optimization problem can be found in the Appendix 9. New (Pareto optimal) strategies can be generated by solving this model. We generate ten Pareto optimal solutions as alternative strategies using this model, as possible ranges of their objective values across all scenarios shown in Figure 6 [2].

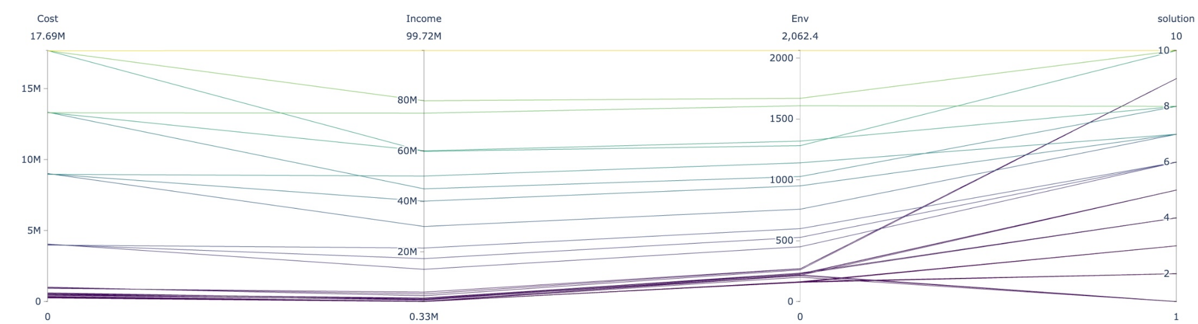

Figure 6: Ten Pareto optimal strategies to the two-stage agrivoltaic farming problem under three scenarios.

`Back to Step 2 for new Antifragility analysis`

To analyze the antifragility scores of these newly generated strategies, we again utilize the proposed measurement corresponding to the same antifragility thresholds set earlier. Table 6 portrayed the number of failures in meeting the antifragility thresholds for each solution regarding each criterion/objective, plus relevant total antifragility scores TAF_{ij} in the last row. As can be seen in this figure, strategy '1' is anti-fragile in all scenarios, proving the identification of at least one anti-fragile strategy. Besides, the total antifragility score TAF_j of all strategies is higher than 0.66, which highlights the antifragility of any strategy based on at least two criteria. Indeed, all the strategies are anti-fragile regarding the environmental benefit criterion and all, but two, regarding the income.

If only these scores are compared, most of the strategies seem fragile in all three scenarios with respect to

[2] Again, the aim was not to generate all Pareto optimal solutions but some diverse samples for illustration and discussion.

Criterion	Strategies									
	1	2	3	4	5	6	7	8	9	10
f_1	0	0	0	3	3	3	3	3	3	3
f_2	0	2	2	0	0	0	0	0	0	0
f_3	0	0	0	0	0	0	0	0	0	0
TAF_j	**1**	0.77	0.77	0.66	0.66	0.66	0.66	0.66	0.66	0.66

Table 6: The fragility and antifragility scores of 10 new strategies (with the possibility of installing an agrivoltaic system) across three scenarios.

the first objective, cost. However, the actual antifragility can be identified by comparing the objective values in Figure 6. Although some strategies (e.g., strategies ‘6´, ‘7’, ‘8’, ‘9’, and ‘10’) have fragile scores based on the set threshold for maximum cost, their outcomes in the other two criterion, income and environmental benefits, are enormously higher than the thresholds and regular performances which is exactly match with the antifragility concept, i.e., thrive and benefit after the change. It means that the extra cost of installing an agrivoltaic system can autonomously boost the business and its benefits, including income and environmental advantages.

`Step 4. Anti-fragile decision-making`

After generating antifragile strategies and conducting initial investigations, decision-makers should conduct a more in-depth analysis, including a trade-off analysis, to choose a (meta-)strategy for their final decision. At this step, decision-makers can, e.g., select some of the generated strategies (from any earlier iteration) to closely investigate their trade-offs between criteria and the uncertainty effects. Figure **??** demonstrates the five strategies (strategies ‘1’, ‘6’, ‘7’, ‘9’, and ‘10’) that have been selected for further analysis. Therefore, the decision-makers can decide the most preferred strategy to be implemented.

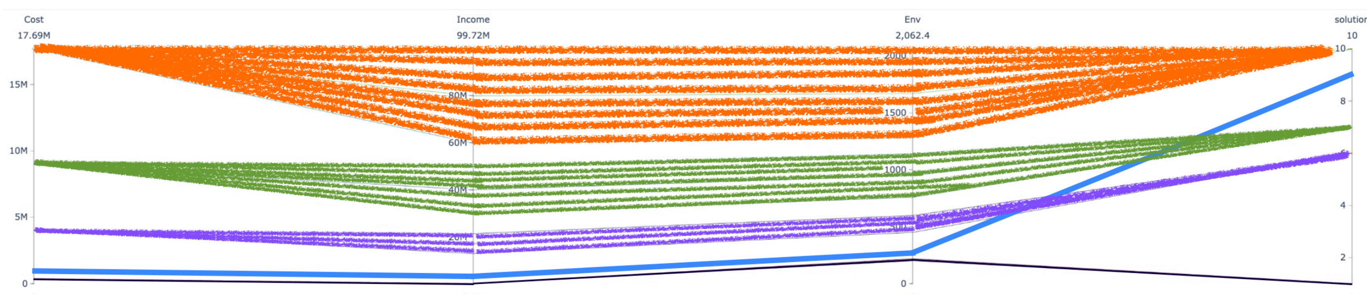

Figure 7: Final five strategies selected for trade-off analysis. The colored shaded areas describe the variation ranges of the objective values (criteria) in different scenarios. See the online version for colors, strategy ‘1’ (■), ‘6’ (■), ‘7’ (■), ‘9’ (■), ‘10’ (■).

For instance, if the initial thresholds cannot be violated and changed, then strategy ‘1´ can be the best option as it is anti-fragile in all scenarios. On the other hand, because of the potential of boosting the income and environmental benefits, if the decision-making can accept the extra cost of installing an agrivoltaic system (e.g., by getting a loan), then choosing any other strategy would be the best option corresponding to accepting the extra cost, i.e., allocating further land to installing the agrivoltaic system. Comparing the value of the first four decision variables (x_1^0, x_2^0, x_3^0, and x_4^0) in strategies ‘1’ and ‘10’ can vividly represent this conclusion, as shown in Table 7. Strategy ‘1’ allocates most of the land for regular farming and a small portion for the agrivoltaic system, whereas strategy ‘10’ allocates all the land to installing the agrivoltaic system and farming behind it. Apparently, planting corn is more beneficial behind the agrivoltaic system (based on y_1^0, y_2^0, and y_3^0) compared to the further suitability of planting soybeans in traditional farming.

Strategy	Decision variables						
	x_1^0	x_2^1	x_3^2	x_4^0	y_1^0	y_2^0	y_3^0
'1'	207	38.2	754.2	0.6	0.35	0.2	0.05
'10'	0	0	0	1000	0	1000	0

Table 7: Comparing the decision variables in strategy '1' and '10'. All the values are in hectares.

8 Discussion

In this section, we summarize and discuss the main essential aspects of anti-fragile decision-making based on findings of the literature review and our proposed measurement and framework.

The first and most important point in complex decision-making is the ability to handle deep uncertainty, when no single possible outcome or a reliable average (or common) case is available and significant changes in various aspects are expected, since relevant problems may well have some kinds of surprises or Black Swans that are unlikely to have perfect predictions and known probability distributions. It is also not always possible to aggregate various possibilities or opt for the most common one, for example, in decision-making in multi-hazard problems.

Multi-criteria evaluation and relevant decision-making support are other vital factors required due to the various conflicting properties and attributes of antifragility, as well as the different aspects of multiple stakeholders, which also reflect sustainability across strategies. Although considering multiple kinds of conflicting performances and relevant decision-making itself is sometimes considered a variety of deep uncertainty (Walker et al. 2013), and uncertainty effects can add an extra complexity dimension, making comparisons and selections challenging, ignoring these crucial and effective factors can lead to failure. Therefore, further support for their consideration, investigation, and representation must be included in relevant decision-support tools.

Additionally, the general issue of aggregating performances across various scenarios or objectives, which can hide even significant compensations, should be considered. Many methods, such as traditional weighted sum and probabilistic methods, focus on some sort of aggregation to reduce complexity dimensions and ease decision-making. However, as discussed earlier, the sole consideration of aggregated outcomes can hide specific failures or opportunities that are compensated for in other criteria or scenarios (Shavazipour et al. 2021b). For instance, aggregating criteria/objectives via the original weighted sum method can fail in identifying potential strategies lying on the non-convex part of the Pareto front of the performance outcomes. Similarly, aggregation across scenarios can hide extremely poor outcomes in a few scenarios, which are offset by small positive outcomes in many other scenarios. Therefore, if that scenario is realized or that criterion becomes important, the decision will easily fail in that case. Thus, informing the decision-makers about such situations is one of the duties of a decision-support system. Of course, then the decision-makers can make a decision based on their risk profile.

On the other hand, anti-fragile strategies typically prioritize long-term performance over short-term ones, particularly when planning for overall performance. It is in conflict with short-term performances, which are prioritized by most managers to represent their competencies and avoid being dismissed. Of course, both types of performance are important and should be considered in planning to facilitate regular operation, prolonged competition, and the survival of an organization. To this end, having skin in the game, directly affected by both positive and negative outcomes, rather than the case where others will bear the negative effects, can provide balance at such a crossroads and consideration of both.

Moreover, considering a wide range of scenarios and evaluating the consequences of potential strategies on them is crucial for identifying fragilities (Levin et al. 2014). Then, transforming threats into opportunities is a key perspective for improving the system's antifragility. However, it should not come at the expense of others. Therefore, being directly involved in the problem and its consequences (having skin in the game) can also support this fact.

Ultimately, one can develop various strategies by employing different models. When considering, for example, antifragility in a model (as objectives or constraints), the generated strategies should have better values in the measurement metrics compared to those that did not explicitly consider it. Therefore, instead of comparing the generated strategies from different models, the fact of employing an anti-fragile perspective should be emphasized over conventional perspectives whenever it is possible. Indeed, utilizing various models can facilitate further investigations, validation, and analysis, and is even essential for identifying adaptation strategies in certain scenarios, such as multi-hazard problems.

The proposed HYDRA framework addresses all these concerns by not requiring knowledge of scenario probabilities and allowing as many scenarios as possible to be considered, enabling it to deal with deep uncertainty. Moreover, various independent scenarios can be assessed, and the multi-criteria nature proposed for some steps can facilitate multi-criteria evaluation in different scenarios and their application in decision-making problems. In addition, the multi-criteria evaluation nature removes the aggregation issue and possible compensations of performance between different criteria and/or scenarios. Similarly, both short-term and long-term performances, as well as their potential conflicts, can be studied for suitable strategy selection and adaptation. One can also utilize interactive methods to directly involve decision-makers in solving irrelevant multi-criteria or multi-objective problems, highlighting the skin-in-the-game property in HYDRA. Finally, consideration of multi-stage approaches (e.g., Shavazipour & Stewart (2021, 2026)) within the HYDRA framework fosters a wider scenario reflection and a thorough evaluation of the diverse consequences of potential strategies that improve antifragility. HYDRA can also be applied for group decision-making for more sustainable decision-making and extends to use multiple models in different steps for anti-fragile decision-making in more complex problems.

9 Conclusions

After two and a half decades of the twenty-first century, humanity continues to struggle with various crises. The Middle East crises, the war in Ukraine, and recent energy and economic crises, pandemics, and climate change highlight the vulnerability and fragility of decisions made based on unreliable predictions, ignoring plausible but less probable scenarios, and careless consideration of the consequences of those decisions on other stakeholders. Although robust decision-making, sustainability, and resilient plans have become hot topics nowadays, recent worldwide crises accentuate their fragilities from various aspects (lose-lose instead of win-win effects). Thus, it is vital for a paradigm shift in decision-making and investigation of novel patterns to handle deep uncertainty, where the likelihoods of plausible scenarios are unknown, or the experts cannot agree upon one.

In this study, we first systematically reviewed the literature to understand how antifragility is defined, measured, and modeled in decision-making problems across various disciplines. This review led us to identify and classify the effective properties and attributes of anti-fragile systems, as well as relevant measurements. After identifying their limitations and the required properties for a reasonable measurement in decision-making, we proposed a proper antifragility measurement that matches the requirements and possible meaningful ways of its usage in AFDM, such as relevant strategy generation and analysis. Finally, we designed a generic anti-fragile decision-making framework (HYDRA) to determine, analyze, and compare anti-fragile strategies, including adaptive pathways and contingency plans, that are not only resilient to disruptions and potential transformations but also enhance overall performance over time, based on multiple criteria.

Our proposals conceptualize antifragility in general decision-making, following the identified properties and requirements, and propose a promising approach for anti-fragile decision-making. As mentioned earlier, *the most important fact is to accept this different way of thinking and plan for growth and thriving instead of surviving at any price.* However, the possibility of applying to various decision-making problems and the relevant adjustments needs further investigation. Additionally, scenario generation and the feasibility of covering sufficiently large situations need to be validated in various cases, and a more thorough exploration of the uncertainty space is required.

Accordingly, some crucial future research directions can be listed as follows:

- Validation and particular requirement adjustments of the proposed antifragility measurement and generic antifragility framework, HYDRA, with real-world problems, both in expert-based and model-based decision-making.

- Developing iterative scenario generation methods for a better presentation of the uncertainty space and fragility identification.
- More specific model-based approach for adaptive decision-making with details for multi-scenario multi-objective optimization methods, including required adjustments for the problem in question.
- Integrating adaptive pathways techniques and multi-scenario multi-objective optimization methods to generate adaptation/contingency strategies in various scenarios.
- Developing interactive visualizations and user interfaces to better support uncertainty presentations, investigations, and comparison of various aspects of potential strategies and decision-making.
- Designing a practical recommendation for managers to (i) develop an anti-fragile organization, and (ii) develop anti-fragile strategies.
- Developing a human-centric AI-help tool for a wide range of scenario evaluations of many strategies, online monitoring, and explainable adaptation recommendations considering all available and simulated data, as well as data based on expert judgment.

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Appendix

A. Two-stage model for combining an agrivoltaic system and the farming problem in Section 7.

$$MinZ_{11} = 240x_1^0 + 430x_2^0 + 290x_3^0 + 17500x_4^0 + 0.4(240y_1^0 + 430y_2^0 + 290y_3^0), \qquad k = 1;$$

$$MinZ_{12} = 250x_1^0 + 450x_2^0 + 340x_3^0 + 17500x_4^0 + 0.4(250y_1^0 + 450y_2^0 + 340y_3^0), \qquad k = 2;$$

$$MinZ_{13} = 275x_1^0 + 475x_2^0 + 390x_3^0 + 17500x_4^0 + 0.4(275y_1^0 + 475y_2^0 + 390y_3^0), \qquad k = 3.$$

$$MaxZ_{21} = 1600x_{11}^1 + 1250x_{21}^1 + 3050x_{31}^1 - 1800x_{41}^1 - 1400x_{51}^1 + (0.1)(1000000)x_4^0, \qquad k = 1;$$

$$MaxZ_{22} = 1850x_{12}^1 + 1450x_{22}^1 + 3300x_{32}^1 - 2000x_{42}^1 - 1600x_{52}^1 + (0.1)(800000)x_4^0, \qquad k = 2;$$

$$MaxZ_{23} = 2100x_{13}^1 + 1650x_{23}^1 + 3500x_{33}^1 - 2300x_{43}^1 - 1800x_{53}^1 + (0.1)(600000)x_4^0, \qquad k = 3.$$

$$MaxZ_{31} = (0.14)x_1^0 + (0.12)x_2^0 + (0.22)x_3^0 + (2)x_4^0 + (0.4)(1 + 0.3)(0.14y_1^0 + 0.12y_2^0 + 0.22y_3^0), \qquad k = 1;$$

$$MaxZ_{32} = (0.13)x_1^0 + (0.13)x_2^0 + (0.24)x_3^0 + (1.6)x_4^0 + (0.4)(1 + 0.35)(0.13y_1^0 + 0.13y_2^0 + 0.24y_3^0), \qquad k = 2;$$

$$MaxZ_{33} = (0.12)x_1^0 + (0.14)x_2^0 + (0.25)x_3^0 + (1.2)x_4^0 + (0.4)(1 + 0.3)(0.12y_1^0 + 0.14y_2^0 + 0.25y_3^0), \qquad k = 3.$$

s.t.

$$\begin{aligned}
& x_1^0 + x_2^0 + x_3^0 + x_4^0 = 1000 && (\textit{Land limitation})\\
& y_1^0 + y_2^0 + y_3^0 - x_4^0 = 0 &&\\
& 4x_1^0 - x_{11}^1 + x_{41}^1 + (0.4)(4)(1 - 0.05)y_1^0 = 400 &&\\
& (3.6)x_1^0 - x_{12}^1 + x_{42}^1 + (0.4)(3.6)(1 - 0.1)y_1^0 = 400 && (\textit{Wheat requirement})\\
& (3.3)x_1^0 - x_{13}^1 + x_{43}^1 + (0.4)(3.3)(1 + 0.1)y_1^0 = 400 &&\\
& 9x_2^0 - x_{21}^1 + x_{51}^1 + (0.4)(9)(1 - 0.1)y_2^0 = 440 &&\\
& 6x_2^0 - x_{22}^1 + x_{52}^1 + (0.4)(6)(1 - 0.15)y_2^0 = 440 && (\textit{Corn requirement})\\
& 5x_2^0 - x_{23}^1 + x_{53}^1 + (0.4)(5)(1 + 0.05)y_2^0 = 440 &&\\
& (2.2)x_3^0 - x_{31}^1 + (0.4)(2.2)(1 - 0.05)y_3^0 = 0 &&\\
& 2x_3^0 - x_{32}^1 + (0.4)(2)(1 - 0.1)y_3^0 = 0 && (\textit{Soybean balance})\\
& (1.5)x_3^0 - x_{33}^1 + (0.4)(1.5)(1 + 0.15)y_3^0 = 0 &&\\
& x_1^0, x_2^0, x_3^0, x_4^0, y_1^0, y_2^0, y_3^0, x_{1k}^1, x_{2k}^1, x_{3k}^1, x_{4k}^1, x_{5k}^1 \geq 0 && k = 1, 2, 3.
\end{aligned} \tag{5}$$